Geometrical interpretation of the argument of weak values of general observables in CP^{N-1} and on the Bloch sphere

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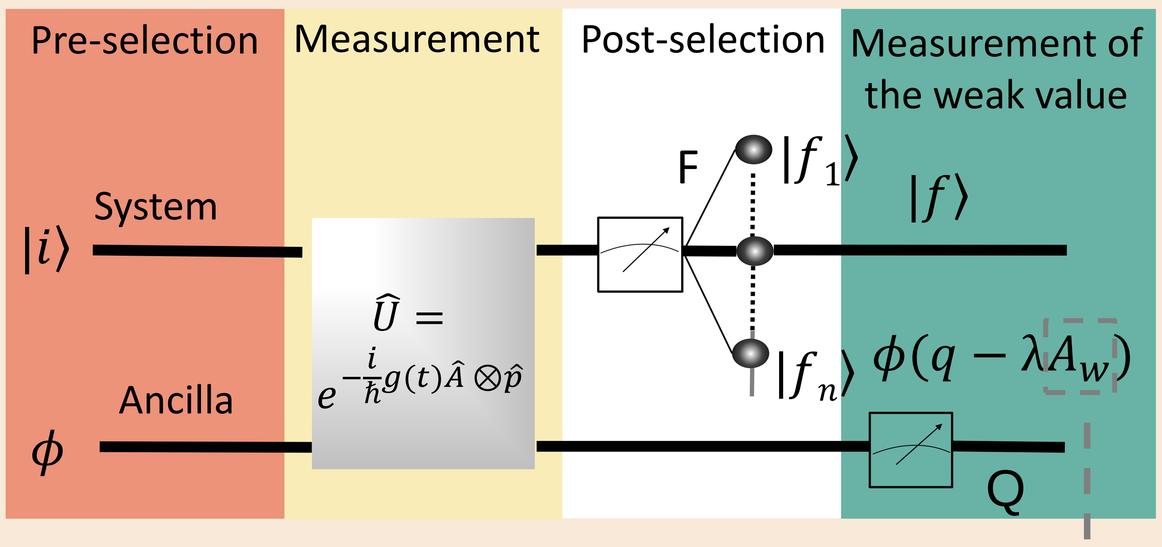
Weak value

Weak values arise in a weak pre- and post-selected von Neumann measurement. The measurement is performed through a unitary operator involving the system and the measuring device. The measuring strength should be very small so that the unitary operator can be developed in series of Taylor.

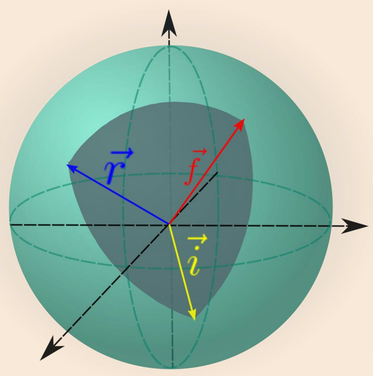


$$A_w = \frac{\langle f|\hat{A}|i\rangle}{\langle f|i\rangle}$$

Imaginary and unbounded number

Argument of the weak value of two-level observables

Argument of the projector weak value



$$\hat{\Pi}_r = |\psi_r\rangle\langle\psi_r| = \frac{1}{2}\left(\hat{I} + \vec{r}\cdot\hat{\vec{\sigma}}\right)$$

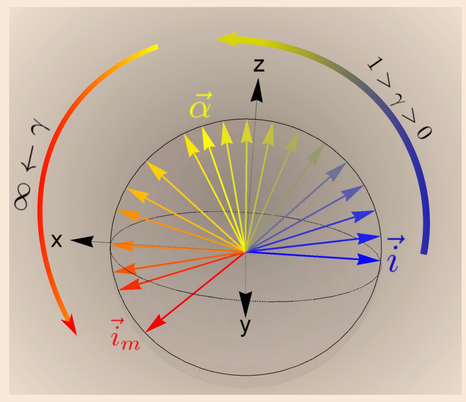
$$\arg\Pi_{r,w} = \arctan\frac{\vec{f}\cdot\left(\vec{r}\times\vec{i}\right)}{1+\vec{f}\cdot\vec{r}+\vec{f}\cdot\vec{i}+\vec{r}\cdot\vec{i}} = -\frac{\Omega_{irf}}{2}$$

Argument of general observable weak value

$$|\psi_{i'}\rangle = \frac{1}{\sqrt{N}}\hat{O}|\psi_i\rangle \qquad i \rightarrow i'$$

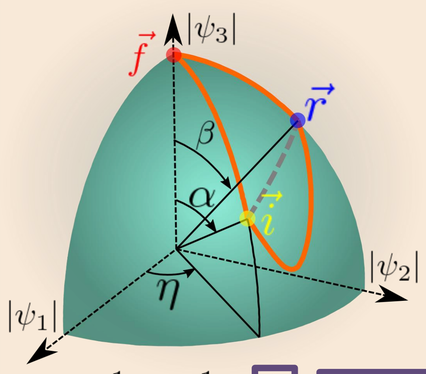
$$\hat{O} = a\left(\hat{I} + \gamma\vec{\alpha}\cdot\hat{\vec{\sigma}}\right)$$

$$\arg O_w = \arg\Pi_{i',w} + \arg\langle O\rangle_i = -\frac{\Omega_{ii'f}}{2} + k\pi \qquad k\epsilon\{0,1\}$$



Argument of the weak value in *N*-level systems

Argument of the projector weak value



The argument of the weak value of a projector in CP^{N-1} describes a geometric phase associated to the symplectic area generated by three vectors representing the pre-selected state, projector state and post-selected state.

$$\hat{\Pi}^r = \frac{1}{N}\hat{I} + \frac{1}{2}\vec{r}\cdot\hat{\vec{T}} \qquad \{T_a,T_b\} = \frac{1}{N}\delta_{ab}\hat{I}_n + \sum_{c=1}^{N^2-1} d_{abc}T_c \qquad [T_a,T_b] = i\sum_{c=1}^{N^2-1} f_{abc}T_c$$

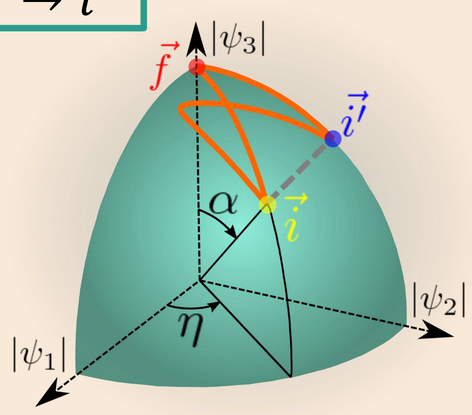
$$arg\left(\Pi_{r,w}\right) = \arctan\frac{\frac{1}{32}f_{abc}f^a r^b i^c}{\frac{1}{N^2}+\frac{1}{8N}\left(\vec{f}\cdot\vec{r}+\vec{r}\cdot\vec{i}+\vec{f}\cdot\vec{i}\right)+\frac{1}{32}d_{abc}f^a r^b i^c}$$

Argument of general observable weak value

$$\hat{A} = a\hat{I} + \vec{n}\cdot\hat{\vec{T}} \qquad A_w = g\Pi_{i',w} \qquad i \rightarrow i'$$

General observable — Real number

The argument of the weak value is associated to the symplectic area generated by the geodesic triangle spanned by $\vec{i}$, $\vec{i'}$ and $\vec{f}$.



Argument of the weak value using the Majorana representation

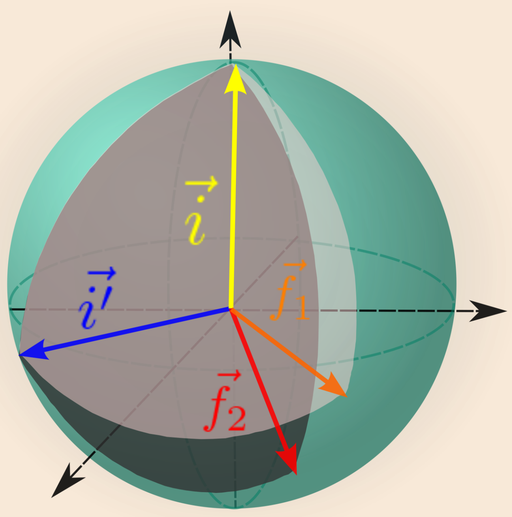
The Majorana representation maps an *N*-level system to *N-1* stars on the Bloch sphere.

Argument of the projector weak value

$$\arg\Pi^{(N)}_{r,w} = \arg\Pi_{1,w} + \arg\Pi_{2,w} + \ldots + \arg\Pi_{N-1,w}$$

$$\arg\Pi^{(N)}_{r,w} = -\frac{\Omega_{irf_1}}{2} - \frac{\Omega_{irf_2}}{2} - \ldots - \frac{\Omega_{irf_{N-1}}}{2}$$

Argument of general observable weak value



$$\arg A_w = -\frac{\Omega_{ii'f_1}}{2} - \ldots - \frac{\Omega_{ii'f_{N-1}}}{2} + \arg\langle A\rangle_i$$

Acknowledgments

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