Discriminating three mirror symmetric states with restricted contextual advantage

Sumit Mukherjee, Shivam Naonit & A.K.Pan

Abstract

- The generalized notion of non-contextuality provides an avenue to explore the fundamental departure of quantum theory from a classical explanation.
- For two state discrimination contextual advantage is available irrespective of any prior probabilities.
- For the case of three mirror-symmetric states, the contextual advantage can be revealed only for a restrictive range of prior probabilities with which the states are supplied
- For maximum confidence discrimination We demonstrate that the prior probabilities of state preparation play a similar role in exploiting the quantum advantage in maximum confidence discrimination.

: Let's see some details

Basic tools

Quantum state discrimination strategies:

- Minimum error state discrimination (MED)
- unambiguous discrimination (UD)
- maximum confidence discrimination (MCD)

Ontological models:

$$\int_\Lambda \mu(\lambda|\rho, P)\xi(k|\lambda, M)d\lambda = Tr(\rho E_k). \quad (1)$$

Does operational equivalence imply ontological equivalence?

Answer: ***Let's assume yes***

Preparation noncontextuality (PNC):
$p(k|P_0, M) = p(k|P_1, M) \Rightarrow \mu_{P_0}(\lambda|\rho) = \mu_{P_1}(\lambda|\rho)$, $\forall M$.

PNC bound of MCD of two states supplied with equal prior probability:

$$S_\lambda^2 \leqslant 1 - \frac{1}{2}c^{\psi_1,\psi_2} \quad (2)$$

Result 1 : Generalization

MED of two states with Arbitrary prior probabilities:

PNC bound:

$$\begin{aligned} S_\lambda^2 &\leqslant 1 - pc^{\psi_1,\psi_2} && for \quad 0 \leqslant p \leqslant 1/2 \\ S_\lambda^2 &\leqslant 1 - (1-p)c^{\psi_1,\psi_2} && for \quad 1/2 < p \leqslant 1 \end{aligned} \quad (3)$$

Quantum bound:

$$S_Q^2 = \frac{1}{2}\left(1 + \sqrt{1 - 4p_1p_2c^{\psi_1,\psi_2}}\right). \quad (4)$$

with $S \Rightarrow$ Success probability, $p \Rightarrow$ prior probability and $S \Rightarrow$ Success probability and $c^{\psi_1,\psi_2} \Rightarrow$ confusibility.

Proposition: *For MED of two nonorthogonal pure quantum states contextual advantage can be revealed regardless of the prior probabilities with which the states are being supplied.*

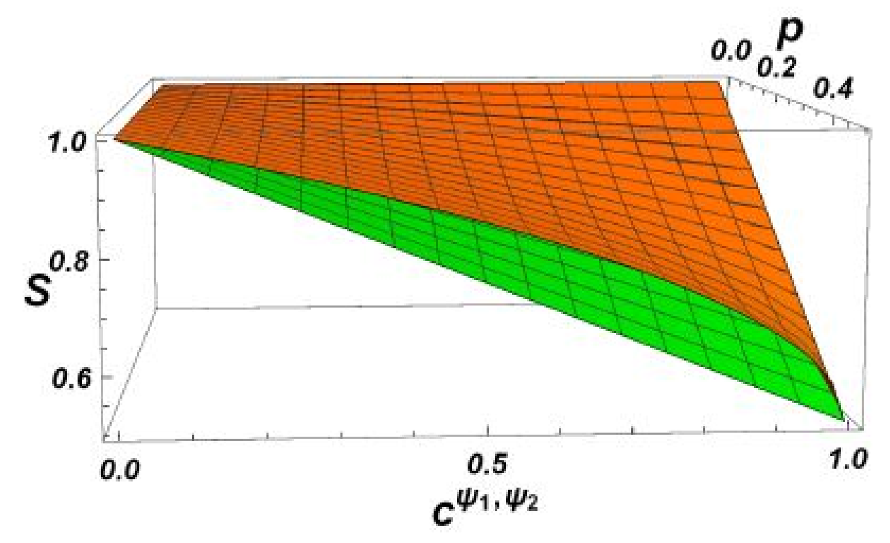

Figure 1: Variation of C with p and θ.

Result 2: Three state generalization

Mirror symmetric states: A transformation $\{\mathcal{T} \in O(\mathcal{H}) \mid \mathcal{T}|0\rangle \to |0\rangle, \mathcal{T}|1\rangle \to -|1\rangle$ leaves the set of states invariant.

$$|\psi_1\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle, |\psi_2\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle, |\psi_3\rangle = |0\rangle, \quad (5)$$

PNC bound:

$$\begin{aligned} S_\lambda^3 &\leqslant 1 - p\cos^2 2\theta - p\cos^2\theta && for \quad 0 \leqslant p \leqslant 1/3 \\ S_\lambda^3 &\leqslant 1 - p\cos^2 2\theta - (1-2p)\cos^2\theta && for \quad 1/3 < p \leqslant 1/2 \end{aligned} \quad (6)$$

Quantum bound:

For $p \geq 1/[2 + \cos\theta(\cos\theta + \sin\theta)]$, it is

$$S_Q^3 \leq p(1 + \sin 2\theta), \quad (7)$$

while for $p \leq 1/[2 + \cos\theta(\cos\theta + \sin\theta)]$ we have,

$$S_Q^3 \leq \frac{(1-2p)(p\sin^2\theta + 1 - 2p - p\cos^2\theta)}{(1 - 2p - p\cos^2\theta)}, \quad (8)$$

Theorem 1: *For MED of three mirror-symmetric states, the contextual advantage for a certain value of confusability can be revealed for a restrictive range of the prior probabilities.*

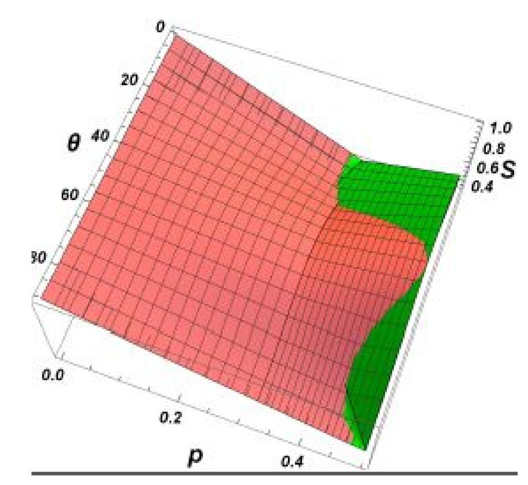

Figure 2: variation of S with p and θ for arbitrary mirror symmetric states.

: **Why?**

Result 3: Maximum confidence strategy

MCD with Arbitrary prior probabilities:

PNC bound:

$$C_\lambda = \left[1 + \cos^2 2\theta + (\frac{1}{p} - 2)\cos^2\theta\right]^{-1} \quad (9)$$

Quantum bound:

$$C_Q = \frac{1 + 2p\cos 2\theta}{2 - 4p\sin^2\theta}. \quad (10)$$

$C \Rightarrow$ figure of merit or the Maximum confidence.

Theorem 2: *For maximum confidence discrimination of three arbitrary mirror-symmetric states, the contextual advantage is available only for a restricted range of prior probabilities.*

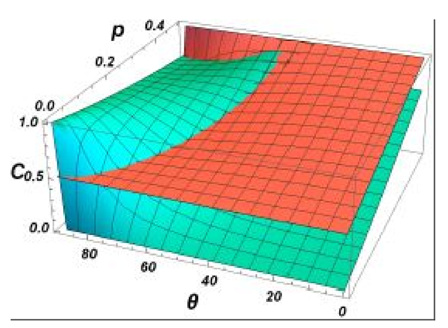

Figure 3: Variation of S with p and θ.

References

1. D. Schmid and R. W. Spekkens, **Contextual Advantage for State Discrimination, Phys. Rev. X 8, 011015 (2018)**
2. S. Mukherjee and S. Naonit, A.K. Pan, **Discriminating mirror symmetric states with restricted contextual advantage, https://doi.org/10.48550/arXiv.2112.15452**