State retrieval beyond Bayes' retrodiction and reverse processes

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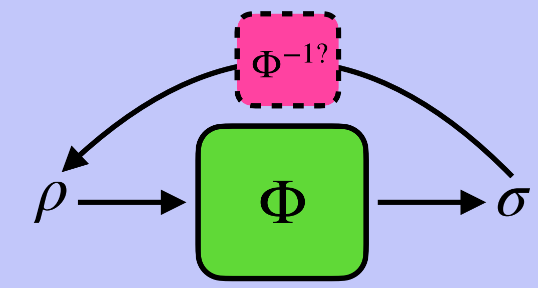
Reverse processes



You try inverting the channel Λ, but unfortunately…

You discover that reversing a channel is quite an ambiguous task. The lack of bijectivity or surjectivity, the fact that channels are contractive, all this adds up to creating more ambiguity.

What do you do?

Reference

https://arxiv.org/abs/2201.09899

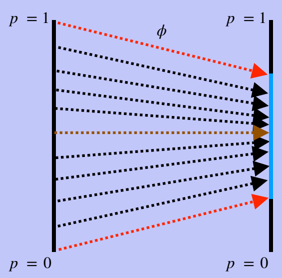
Commonly used reverse processes

An illuminating approach is adopting a statistician perspective and associating reverse processes with the process of retrodiction. It has been shown in (2009.02849) that the common method for defining a generalised reverse map is analogous to the operation of retrodiction based on Bayes' theorem.

Classical probability: probability vectors $\vec{p}$ and stochastic matrices Φ — $(\tilde{\Phi}_B)_{i,j} = \Phi_{j,i}\pi_i/(\Phi(\pi))_j$ — Bayes inspired reverse map

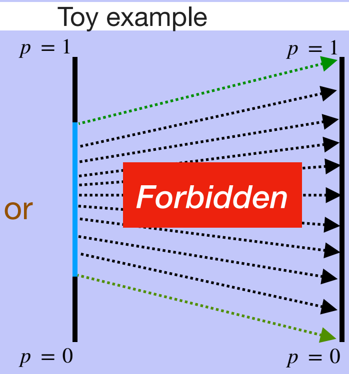
Quantum mechanics: quantum states $|\psi\rangle$ and CP channels Φ — $\tilde{\Phi}_P = \mathbb{J}_\pi \Phi^\dagger \mathbb{J}^{-1}_{\Phi\pi}$, — Petz map (Petz recovery map) $\left(\mathbb{J}_\pi(\rho) := \sqrt{\pi}\rho\sqrt{\pi}\right)$

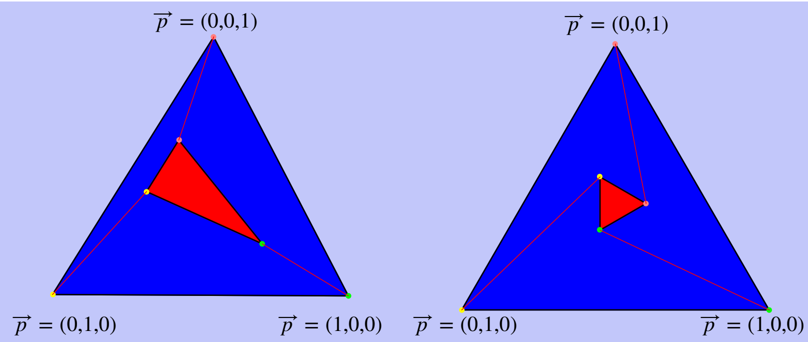
Toy example



To keep in mind:

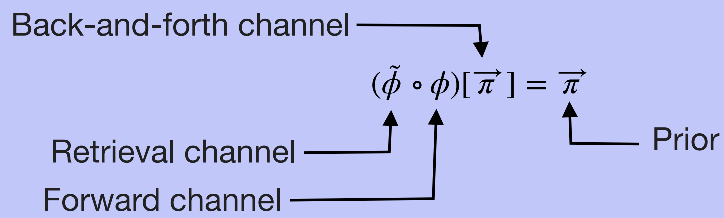
- Channels "do not expand"
- Channels always preserve at least one vector or state, they have at least one fixed point





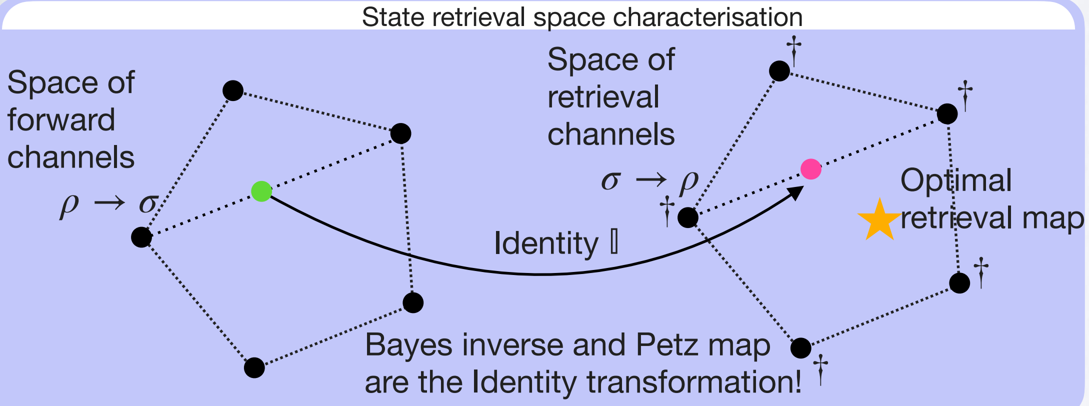
Characterisation of state retrieval maps

1. **If the channel can be inverted just by simply inverting the arrow, just do it.**
"we already know how to take the inverse of a permutation or unitary channel".
2. **The state retrieval channel should be physical.**
"It should be a meaningful retrieval map even in the single-shot scenario, not just in the full statistics case".
3. **The fixed point of the back-and-forth channel must be the prior.**
"Since every channel has a fixed point, we take advantage of this property and we encode all our additional knowledge in it. We select a typical initial state that we want to always perfect recover".
4. **The prior should be an equilibrium state for the back-and-forth channel.**
"The back-and-forth channel should be as static as possible, at least in the prior".
5. **All the eigenvalues of the back and forth map must be positive.**
"Every inversion (negative eigenvalues) or rotation (complex eigenvalues) ruins the retrieval."



$$(\tilde{\Phi}\circ\Phi)_{j,i}\pi_i = (\tilde{\Phi}\circ\Phi)_{i,j}\pi_j$$

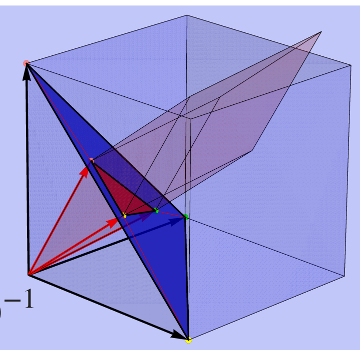
State retrieval space characterisation



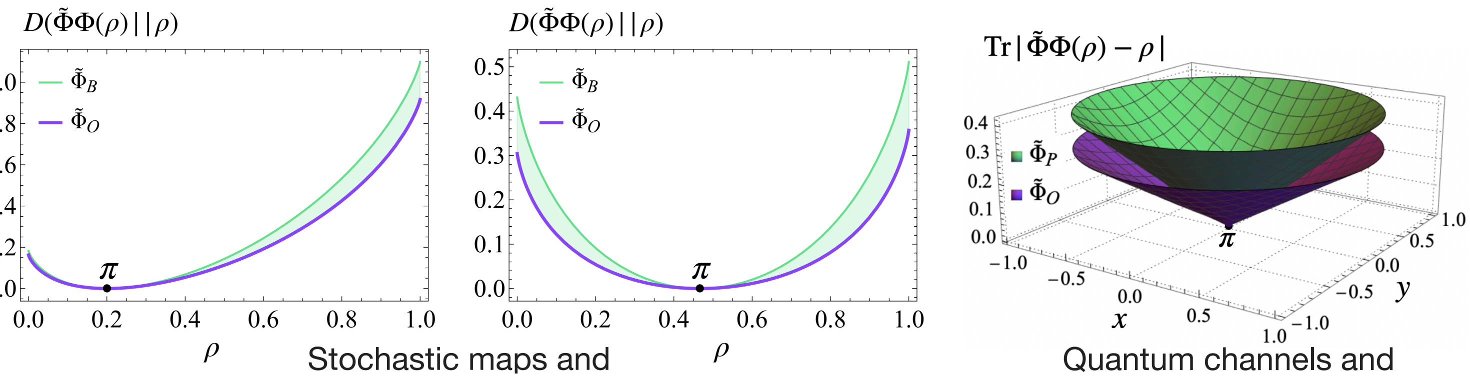
Optimal retrieval map

$$\tilde{\Phi}_O = \max_{\tilde{\Phi}\ \text{state retrieval}} \det \tilde{\Phi}\Phi$$

$$D(\tilde{\Phi}\Phi||\mathbb{1}) = \mathrm{Tr}[\mathbb{1}(\log\mathbb{1} - \log\tilde{\Phi}\Phi)] = -\mathrm{Tr}[\log\tilde{\Phi}\Phi] = \log\det(\tilde{\Phi}\Phi)^{-1}$$



Comparison of the state retrieval with Bayes and Petz



Stochastic maps and

Quantum channels and

Recovering Bayes and Petz

Can we add a property that isolates Bayes and Petz?

Maybe. Numerical evidence of a sufficient 6th property (involutivity) that isolates Bayes and Petz.