Improved universal hypergraph resource state for MBQC with Pauli X and Z measurements

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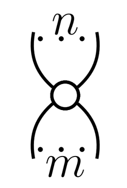
Introduction

Measurement-based Quantum Computation (MBQC) is a universal model for quantum computation. MBQC involves the preparation of a resource state followed by adaptive stepwise measurements, which can perform the computation of our choice. Usually, the resource state is a graph state, and universality requires measurements from various bases, including non-Pauli measurements [1].

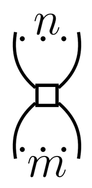
We study universal hypergraph states which offer universality with only Pauli X and Z measurements. The first such state was presented in [2], and it requires $\Theta(n^4 d)$ qubits to perform a circuit on n qubits of depth d. Here, we present a different variant that requires $\Theta(n^2 d)$ qubits. We discuss how the state can be deterministically measured by utilizing phase-free ZH Calculus [3].

Phase-free ZH

The phase-free ZH Calculus is a universal and complete graphical language for quantum computation. It consists of string diagrams constructed from generators (called spiders) and a set of rewrite rules explaining how the diagrams can be transformed [3]. The generators are:

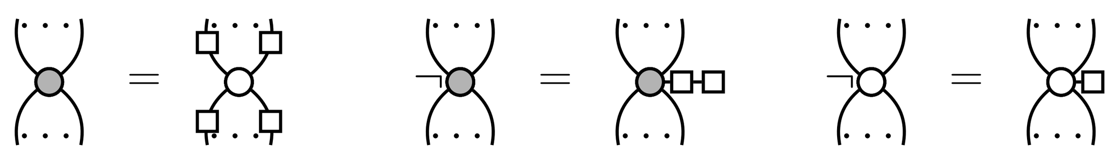


$\rightsquigarrow |0\rangle^{\otimes n}\langle 0|^{\otimes m} + |1\rangle^{\otimes n}\langle 1|^{\otimes m}$



$\rightsquigarrow \sum (-1)^{i_1\ldots i_m j_1\ldots j_n} |j_1\ldots j_n\rangle\langle i_1\ldots i_m|$

There is also a star generator which represents a scalar $\frac{1}{2}$. However, we will omit scalars here. A few derived generators are useful when representing MQBC schemes:



The white spider corresponds to ZX green spider, the dark spider to ZX red spider, and the NOT symbol to π phase from ZX. A dashed blue line represents a wire containing a box with two legs, similar to the notation from ZX Calculus.

Measurement-based Quantum Computation

The ZX Calculus can efficiently represent computations performed in the MBQC model. Here, we work with resource states that are hypergraph states [4], which happen to give smaller diagrams in ZH.

A hypergraph resource state is obtained from a hypergraph with chosen sets of input and output vertices. For each non-input vertex in a hypergraph, a qubit is prepared in a $|+\rangle$ state. For each hyperedge, a generalised CZ gate is applied to the corresponding qubits. Generalised CZ gates correspond to multi-legged boxes from ZH. An example of hypergraph state is presented in figure 1.

Figure 1. (a) An example of a hypergraph state, following MBQC notation from [5]. Shaded triangles stand for 3-hyperedges. (b) ZH representation of this hypergraph state. Dangling edges to the left and right mark input and output qubits respectively. The measurement effects can be represented by putting spiders at the end of vertical dangling edges.

Universality with Pauli Z and X measurements

The work [2] introduces a state G_n^d, which allows execution of any circuit on n qubits of depth d, constructed with approximately universal [6] gate set H and CCZ.

The key concept, which is also used here and in [7], is a block that can perform a CCZ gate on the 3 input qubits. We present an example of such a block in 2.

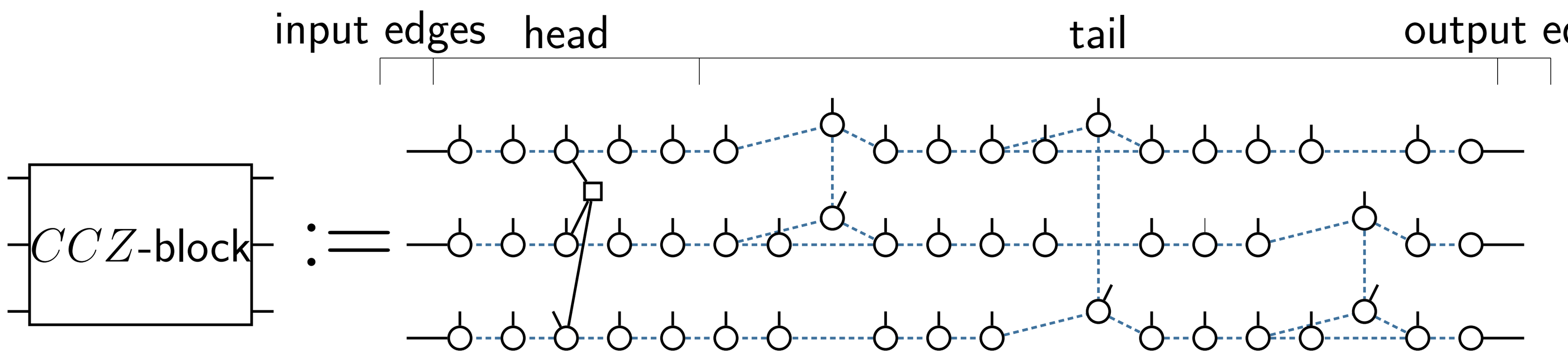


Figure 2. CCZ block. The head is measured in X basis, and the tail is adaptively measured in X or Z in such a way that any two-qubit byproducts obtained from measuring the head are corrected.

New universal resource state

Depending on how the CCZ blocks are connected, we may obtain versions equivalent to G_n^d with various numbers of qubits. Initial work required a block for any possible triple of the qubits. By observing that at most $\lfloor \frac{n}{3} \rfloor$ CCZ gates can be applied in any circuit of depth one, we can reduce the total number of qubits.

By stacking the blocks vertically, we allow parallel execution of the gates. An overview of the improved state (called $\tilde{G}_n^1$) is presented in figure 3.

The extension to depth d remains the same as in the original work and [7] – it is a sequential connection of d copies of $\tilde{G}_n^1$.

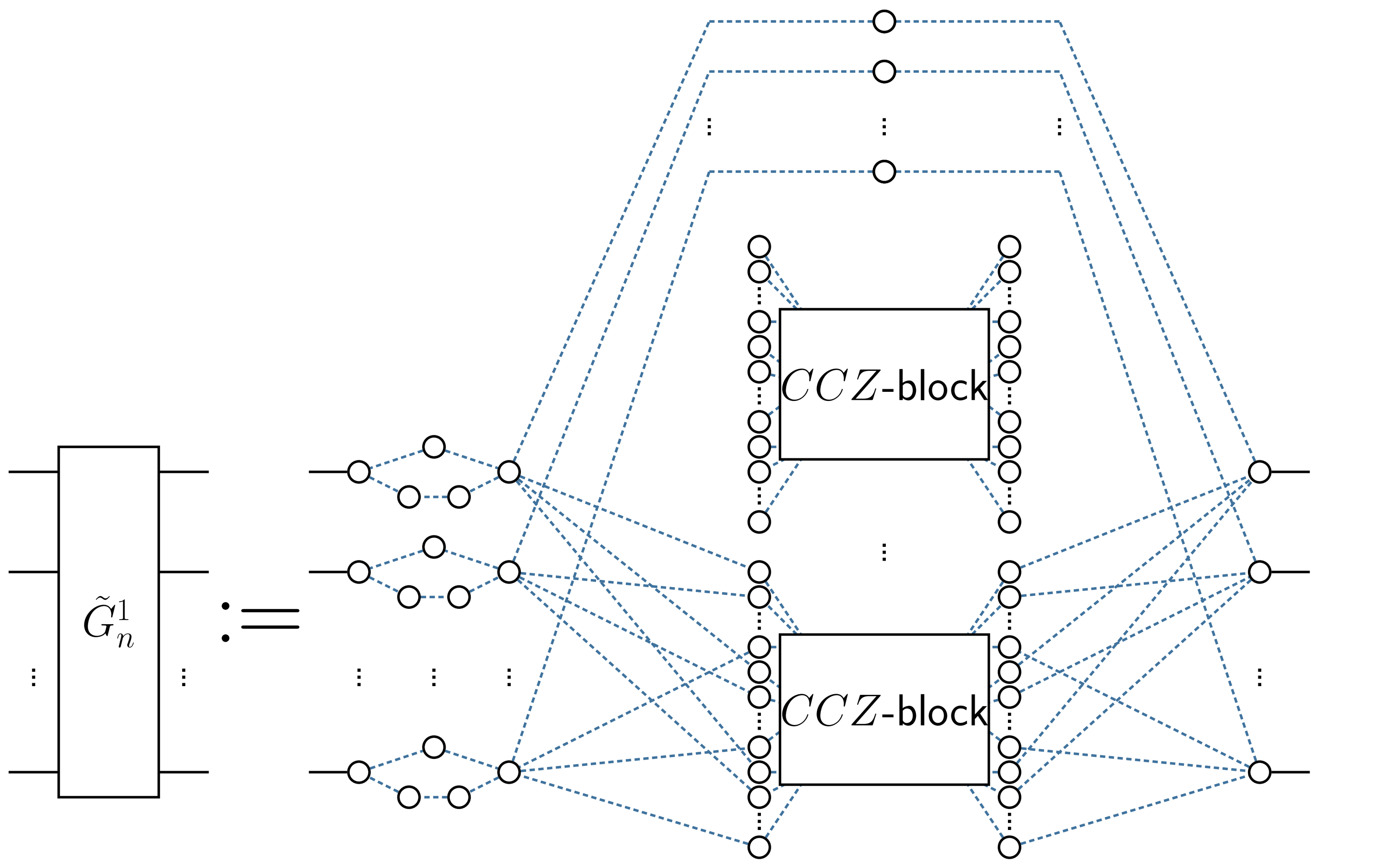


Figure 3. $\tilde{G}_n^1$ state – an improved version of the G_n^1 state. There are $\lfloor \frac{n}{3} \rfloor$ CCZ blocks. Otherwise, ⋮ indicates that a structure is repeated n times. To make the diagram more readable only connections for one of the CCZ-blocks are shown. For the same reason, vertical dangling edges are not included.

Measurements and correction of byproducts

Each measurement has two possible outcomes, but only one is desired. Undesired measurements are represented in ZH by NOT spiders. If the measurement result is undesired, we must adapt future measurements to modify the state to one that would be obtained in case of desired measurement outcome. In graph state MBQC, this relies on the application of Pauli gates to unmeasured qubits. With only Pauli measurements, this process is redundant. However, we may obtain two-qubit byproducts (figure 4), and fixing these may require changing measurement bases.

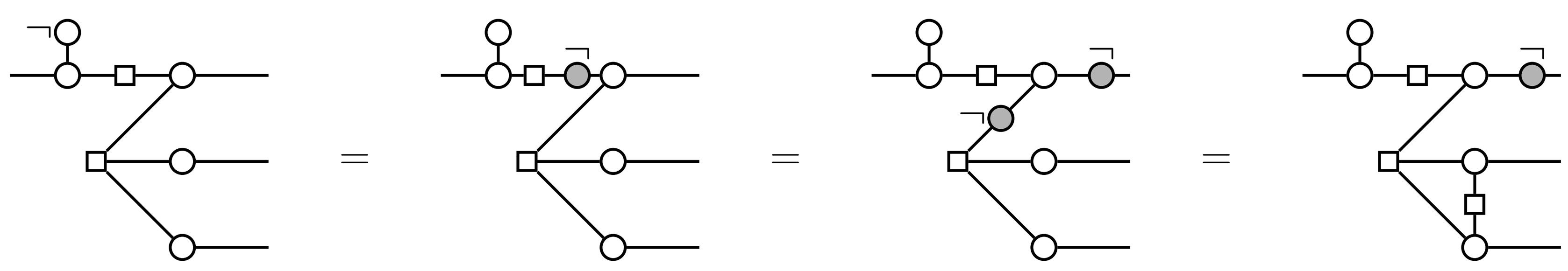


Figure 4. Pushing dark NOT through a box with 3 legs results in a two-qubit byproduct – the final diagram contains extra edge between two of the output spiders.

Byproducts are fixed based on the following facts:

- All qubits in a hyperedge of degree 3 are measured in the same bases.
- For each qubit measured in the Z basis there is precisely one qubit to the left measured in the Z basis and one qubit to the right measured in the Z basis.
- Application of a Pauli X or Z gate to a yet unmeasured qubit corresponds to either flipping the result of future measurement or simply doing nothing.

Future work

The second fact above is reminiscent of the causal flow known for graph state MBQC [8]. It may be interesting to research if a flow structure for hypergraphs can be established. Such a structure might extend causal flow, or one of the other flow structures known for graph states [9, 10, 11].

Unknowingly to the author at the time of preparation of the poster, an improved state has already been presented in [7], and it requires only $\Theta(n \log^2 nd)$ qubits. The improvement utilises sorting networks. However, we can improve the state further, for instance, by encoding sorting networks more efficiently. Furthermore, ZH allows the derivation of simple instructions for correcting undesired measurement outcomes, which are left implicit in the literature.

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