How typical is contextuality?

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Identifying when observed statistics cannot be explained by any reasonable classical model is a central problem in quantum foundations. A principled and universally applicable approach to defining and identifying nonclassicality is given by the notion of generalised noncontextuality. Here, we study the *typicality* of contextuality — namely, the likelihood that randomly chosen quantum preparations and measurements produce nonclassical statistics. Using numerical linear programs to test for the existence of a generalised-noncontextual model, we find that contextuality is fairly common: even in experiments with only a modest number of random preparations and measurements, contextuality arises with probability over 99%. We also show that while the typicality of contextuality decreases as the purity (sharpness) of the preparations (measurements) decreases, this dependence is not especially pronounced, so contextuality is fairly typical even in settings with realistic noise. Finally, we show that although nonzero contextuality is quite typical, quantitatively high *degrees* of contextuality are not as typical, and so large quantum advantages (like for parity-oblivious multiplexing, which we take as a case study) are not as typical. We provide an open-source toolbox that outputs the typicality of contextuality as a function of tunable parameters (such as lower and upper bounds on purity and other constraints on states and measurements). This toolbox can inform the design of experiments that achieve the desired typicality of contextuality for specified experimental constraints.

This is an extended abstract for arXiv:2510.20722. All proofs and technicalities can be found there.

Preliminaries

Certain quantum experiments exhibit correlations that are incompatible with any classical explanation [1–4]. A gold standard method for demonstrating that a given theory resists to such a classical explanation is assessing generalised contextuality [4–9]. Studies characterizing exactly which phenomena constitute proofs of generalised contextuality have been carried out in several contexts [10–44]. Given the ubiquity of contextuality proofs in a wide range of quantum tasks, a natural question is: how often does contextuality "accidentally" arise in an experiment? Could it be that nearly any experimental quantum setup is hiding some sort of nonclassical resource? We investigate these questions by performing analytical and numerical explorations of contextuality scenarios where states and/or measurements are chosen randomly. More specifically, we aim to answer the following questions:

1. How typical is it to stumble upon contextuality when we randomly sample a finite number of pure states and projective measurements? How does this typicality change with the number of states and the number of measurements?
2. How does the typicality of contextuality depend on whether the states are pure or not and on whether the measurements are projective or not?
3. If one typically stumbles upon contextuality, does one also typically stumble upon a useful resource for quantum advantage?

Methods. We perform extensive simulations of contextuality certification over qubit systems, given by the linear program in Ref. [45]. We characterize each simulation by a tuple of parameters $(n,m,d;N)$, where n is

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the number of preparations and m is the number of binary measurements that comprise the prepare-and-measure scenario, d is the dimension of the Hilbert space associated to the quantum system being investigated, and N is the number of sampling iterations and assessments of contextuality. We randomly sample n density operators and m pairs of POVM elements by employing numerical tools from `numpy` and `qutip` [46], and then employ the linear program to assess the contextuality of the sample. We define the *typicality of contextuality*, denoted by $t_{(n,m,d;N)}$, as the percentage of samples (out of N) that exhibit contextual behaviour. We also investigate scenarios for which the measurements are not random, but instead are fixed and distributed over the Bloch ball. Additionally, we obtain analytical proofs from the topological features of the Bloch sphere, as well as from the geometrical representation of noncontextual ontological models as simplex-embeddability [9]. When referring to the theoretical (ideal) typicality, the N index is suppressed (i.e., we write $t_{(n,m,d)}$).

We also investigate how typicality of contextuality is connected to the success rate in parity-oblivious multiplexing task [28]. For this, we fix the preparations to be those that achieve maximum advantage on a 3-to-1 parity-oblivious task, and replace the optimal measurements with randomly sampled ones. We then compute the typicality of contextuality, alongside the average success rate and the respective deviations for different families of sampled measurements.

Summary of results

We hereby summarise our main results:

Necessary number of preparations (analytical). For a qubit, the typicality for n randomly sampled states and m randomly sampled measurements is 0% as long as $n \leq d^2$. This stems from results in the literature [47] and sets a lower bound on the required number of randomly sampled preparations to witness contextuality.

Typicality is never truly 100% (analytical). We demonstrate that for any finite n and m, $t_{(n,m,d)} < 100\%$. This implies that any numerical instances of $t_{(n,m,d;N)} = 100\%$ should not be interpreted literally, but rather as an asymptotic value. We employ the Wilson score interval [48] to obtain the intervals of values with a sufficient degree of confidence for the numerical maxima.

Typicality converges to 100% as either n or m go to ∞ (analytical). This holds for pure states and sharp measurements, and for the cases not ruled out by our first result (i.e., $n > d^2$). It follows from this result that, for any typicality scenario with finite (but sufficient) n and m, it is possible to find both $m' > m$ and $n' > n$ such that $t_{(n',m,d)} > t_{(n,m,d)}$ and $t_{(n,m',d)} > t_{(n,m,d)}$.

Typicality goes to ≈ 100% faster for pure states/measurements (numerical). In our results, we repeatedly observe how faster typicality converges to very large values when sampling pure states and sharp measurements. A visualisation of such contrast is provided in Figure 1.

"Thin foil" of purity/sharpness (numerical). Because typicality converges much faster for pure states and sharp measurements, one might ask whether this is constrained to the ideal case, or if it is possible to observe it in experiments where maximal purity and sharpness are not attainable. We observe that, as long as the purity/sharpness lies above a certain threshold, the number n for a fixed m required for $t_{(n,m,d;N)} > 99\%$ does not change drastically.

Moreover, we provide an open-source numerical tool where one can assess the sufficient number of preparations n for any dimension d, number of measurements m, and respective bounds in purity/sharpness.

Additionally, we connect and compare our findings to some well-established results:

Typicality of contextuality reaches ≈ 100% faster than typicality of nonlocality (numerical). This is observed by directly comparing our numerical results to the ones in Ref. [49] and suggests that, for performing nonclassical experiments with comparable resources, it is advantageous to probe contextual features rather than nonlocal ones: besides not requiring space-like separation or the preparation of bipartite entangled states, tests of

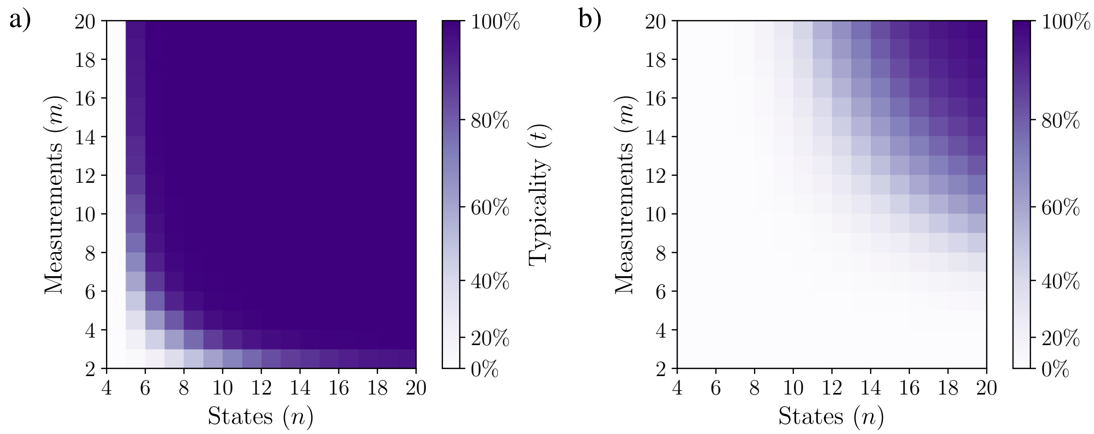

Figure 1: Typicality of contextuality for different numbers of qubit preparations and measurements for randomly-sampled (a) pure states and projective measurements, and (b) mixed states and POVMs.

contextuality can use fewer (or less stringently chosen) procedures to generate nonclassicality.

High typicality does not necessarily translate into high quantum advantage (numerical). We illustrated this by analysing parity-oblivious multiplexing tasks (POM) [28]. By adding constraints to how we sample measurements in POM tasks, we can consider how different obstacles to experimental implementation may affect the ability to obtain nonclassical experiments, as well as how these may affect the ability to obtain advantage. We show, for instance, that reference frame misalignment between the encoding and decoding parties still enables an advantage in this task, although it is significantly reduced compared to the optimal strategy.

Final remarks

In this work, we begin to investigate the question of how typical contextuality is in prepare-and-measure scenarios. In this setting, we find that contextuality is quite typical even for relatively small numbers of states and measurements, and we show how these numbers must increase in the presence of noise. This novel investigation is only possible thanks to efficient numerical tools introduced in recent years [45]. Our study, moreover, goes beyond what has been done to study typicality of nonlocality in that we probe scenarios with mixed states and unsharp measurements. Our results also suggest a useful strategy for engineering contextuality: one should aim to generate as many operational equivalences as possible; for example, ensuring that an experiment spans only a subspace of the state/effect space can significantly enhance the odds of observing contextuality. We provide an open-source toolbox [50] that may guide both analytical investigations and experimental tests of contextuality.

Some new avenues open up with this kind of investigation. For instance, it would be interesting to explore how our methods can be applied to the study of nonclassicality of individual processes, as initiated in Refs. [47, 51]. Considerably more work could be done to study the typicality of nonclassicality of single processes by continuing in this direction. Moreover, it would be interesting to see how typicality of contextuality behaves in more general frameworks (such as the one of generalised probabilistic theories) and how these numerical tools may be employed to investigate novel notions in the contextuality program [52].

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