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Algebraic techniques for photonic states preparation and characterization

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Multimode multiphoton states are at the center of many photonic quantum technologies, from photonic quantum computing to quantum sensing. They also appear, under the name of *core states*, as the central non-Gaussian cores of generic non-Gaussian states of finite stellar rank, for which they also encode the non-Gaussian quantum correlations. These are usually split into two classes. Since any Gaussian state can be brought into a separable one using only interferometers, which preserve the total energy, we say that a generic multimode state is *passively separable* if it can also be brought into separable form using passive linear optics. States that are not passive separable possess *mode-intrinsic* entanglement, since (part of) their entanglement persists in every mode basis. On the other hand, unlike Gaussian states, some non-Gaussian states may be mapped into separable ones using active Gaussian operations (including squeezing), but not just with linear optics. These are called *Gaussian-separable* states, whereas their complement defines *truly non-Gaussian entangled* states. For core states, it is known that either they are passive separable or they are truly non-Gaussian entangled: in other words, they never require active Gaussian operations to be disentangled.

Despite their wide use and apparent simplicity, the vast majority of core states require ad-hoc procedures to be prepared and their entanglement properties remain poorly understood; in particular, it is known that only a measure-zero subset of them can be prepared starting from a multimode Fock state and using only interferometers to entangle the modes [1]. Recently, in the context of continuous-variable quantum information and in the pursuit of a classification for non-Gaussian states, the very versatile tool of stellar polynomials [2, 3] was introduced to describe such states with functions on phase-space. In these works, we take full advantage of this description by showing that the algebraic properties of these multivariate polynomials are related to preparation procedures for the corresponding states and they also characterize the type of entanglement between the modes [4].

The first central result that we present consists in a full characterization of their entanglement structure through the atomic decomposition of their stellar polynomial and its associated structural graph, whose connected components determine the *mode-intrinsic entanglement* of the state and all partitions compatible with passive separability, i.e. the bipartitions into sets of modes that can be disentangled via interferometers. An essential ingredient in this construction is the concept of *essential variables*, which identify the minimal number of effective modes involved in a core state, in direct correspondence with the symplectic rank.

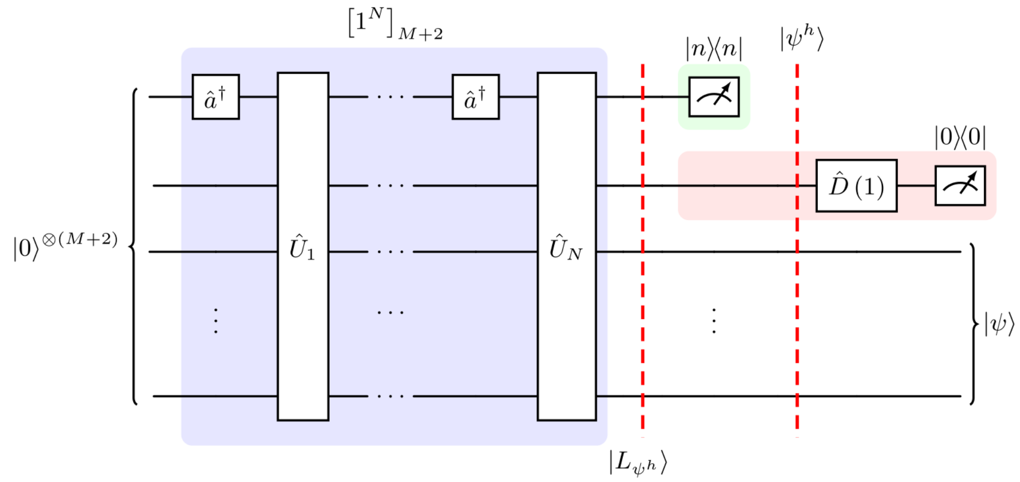

Figure 1: Circuit of the proposed method. By injecting N photons, n of which will be later retrieved by a photon counter (catalyzing photons), we can generate any pure state $|\psi\rangle$ with at most $d = N - n$ photons in M modes, provided that n is sufficiently large. The blue part represents the preparation of the seed state, the stellar polynomial of which factorizes into a product of linear terms, the green part represents the post-selection that allows to construct an arbitrary homogeneous state and the red part represents de-homogenization.

This reduction provides the foundation for decomposing stellar polynomials into atomic factors and for revealing the underlying entanglement structure. Building on this, we derive complete separability criteria for two-mode states, expressed through hyperplane decompositions of zero sets, and for stellar-rank-2 states across arbitrary number of modes. Importantly, this discussion generalizes to a wide class of non-Gaussian pure states, namely those having finite stellar rank, such that our results can be seen as one of the first landmarks in the full characterization of non-Gaussian quantum correlations. Applications to several example states illustrate how the method isolates genuinely non-Gaussian resources and quantifies preparation complexity.

Furthermore, we describe a new procedure [5] to generate exactly, and with a predictable number of steps, any such state by using only interferometers, photon number resolving detectors, photon additions and displacements. We achieve this goal by establishing a connection between stellar polynomials and the problem of symmetric tensor decomposition, for which we could apply known results from algebraic geometry. This unveils a mechanism of *photon catalysis*, where extra photons are injected and subsequently retrieved in measurements, to generate entanglement that cannot be obtained through Gaussian operations acting solely on the photons that we want in the target state. We also introduce another tensor decomposition that generalizes our method and allows to construct circuits yielding perfect fidelity, using the minimum number of catalysis photons. As a benchmark, we numerically evaluate our method and compare its performance with state-of-the art results, confirming 100% fidelity on different classes of states. Moreover, at a difference with previous results, our *de-homogeneization* step allows us to prepare superpositions with non-constant number of photons with a negligible extra cost. Remarkably, by replacing ideal photon additions with realistic protocols involving either beam-splitters with ancillary input modes prepared in a single-photon state or two-mode squeezers with vacuum ancillary modes, our scheme can be mapped into instances of boson sampling or Gaussian boson sampling, respectively.

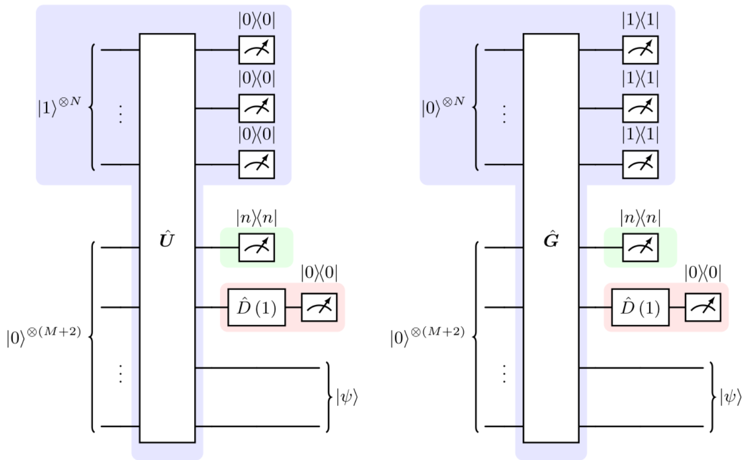

Figure 2: Alternative representations of the proposed method, where approximate photon additions are implemented through injection of single photons coupled by weakly reflecting beam-splitters as in a boson sampler, or by weak single-mode squeezing operations followed by heralding of single photons as in a Gaussian boson sampler.

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Papers associated with the submission

- A. Aralov, É. Gillet, V. Nguyen, A. Cosentino, M. Walschaers, and M. Frigerio, *Photon catalysis for general multimode multi-photon quantum state preparation*, accepted for publication in *PRX Quantum* (2025); available as arXiv:2507.19397 [quant-ph], `https://arxiv.org/abs/2507.19397`.
- C. E. Lopetegui-Gonzalez, M. Frigerio, and M. Walschaers, *Geometric characterization of non-Gaussian entanglement for finite stellar rank states*, arXiv:2511.02076 [quant-ph] (2025), `https://arxiv.org/abs/2511.02076`.