Parity erasure: a foundational principle for indefinite causal order

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1 Extended abstract

The notion of causality, which prohibits signaling from the future to the past, is central to our understanding of the physical world. In quantum physics, processes are conventionally formulated as causal networks in which the input-output connections between devices are interpreted as causal links, and all events are compatible with a well-defined causal order.

Recently, the scope of foundational investigations of causality has expanded to scenarios in which the causal structure of quantum processes is dynamical, showing that quantum theory accommodates more general processes with indefinite causal order (ICO) [1–5]. Such processes are described within the process matrix formalism, in which operations in local laboratories are described by standard quantum theory, while the external environment is characterized by a process matrix: a higher-order quantum operation that generalizes quantum causal networks [2, 3]. They can violate Bell-like inequalities for causal order [3, 4, 6–8], known as causal inequalities. They are expected to arise at the interface of quantum mechanics and general relativity [9, 10] and have also been shown to exist on time-delocalized quantum subsystems in standard quantum mechanics [11, 12].

The possibility of ICO raises fundamental questions about the role of causality in the foundations of physics. Indeed, while a global causal order may be absent, information flow within local laboratories remains causal. Recent work [13, 14] has shown that the quantum formalism imposes nontrivial constraints on correlations with ICO. Yet how such correlations are shaped by causality within local laboratories remains poorly understood — a gap that contrasts with the precise information-theoretic formulation of causality in standard operational probabilistic frameworks [15].

In this work, we formalize the local validity of causality within a locally sequential paradigm, in which we identify fundamental constraints on correlations arising in experiments with locally well-defined causal order. These constraints can be represented by classical higher-order processes and equivalently formulated as a parity-erasure condition for a communication task: certain global parity quantities cannot be transmitted through higher-order quantum processes.

We then demonstrate that the parity-erasure principle fully characterizes higher-order quantum processes at the level of local process-tomography data. Specifically, we show that, among the local process-tomography data of all quantum channels, those corresponding to higher-order quantum processes can be singled out precisely by imposing the parity-erasure condition.

Beyond this characterization, we show that the parity-erasure principle imposes constraints on quantum correlations with indefinite causal order in an operational way that generalizes the method in Ref. [13].

Our results extend to a broad class of causal probabilistic theories satisfying an operational property called one-way-signaling decomposability. In theories that additionally satisfy local tomography, we show that higher-order processes are channels that respect the parity-erasure condition. In theories that additionally satisfy the gentle measurement principle [16], we show that no realization of correlations using higher-order processes can be arbitrarily close to deterministic correlations that are incompatible with classical higher-order processes.

Our results thus provide an information-theoretic axiomatization of the process matrix formalism based on the requirement that causality be preserved locally, revealing a fundamental constraint on information exchange within indefinite causal structures. They offer concrete operational intuition for these otherwise exotic processes and may facilitate the identification of advantages conferred by ICO in quantum information processing. More broadly,

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they suggest that information-theoretic principles may play a central role in any theory with dynamical causal structure, including prospective theories of quantum gravity. They also open the door to a systematic study of ICO in general operational probabilistic theories [15, 17–25] and to a clearer identification of the advantages that processes with indefinite causal order may offer.

An important direction for future work is to explore the implications of the parity-erasure principle for concrete ICO applications, including the characterization of correlations in the presence of ICO [13, 14, 26]. The locally sequential paradigm, which clarifies the causal mechanism of ICO, may provide new methods for device-independent certification of causal nonseparability and the development of cyclic quantum causal models [27–29]. Another natural direction is to relax the assumptions used in this paper in order to better understand the role of causality in physics. In particular, it would be interesting to extend the information-theoretic approach developed here to higher-order processes in non-causal theories, such as bistochastic quantum theory [30] and the multi-round process matrix formalism [31].

2 Link to the paper

Our paper is available on arXiv at https://arxiv.org/abs/2512.08635.

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