The Contextual Heisenberg Microscope

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The Heisenberg microscope provides a powerful mental image of the measurement process of quantum mechanics (QM), attempting to explain the uncertainty relation through an uncontrollable back-action from the measurement device. However, Heisenberg's proposed back-action uses features that are not present in the QM description of the world, and according to Bohr not present in the world. Therefore, Bohr argues, the mental image proposed by Heisenberg should be avoided. Later developments by Bell and Kochen-Specker shows that a model that contains the features used for the Heisenberg microscope is in principle possible but must necessarily be nonlocal and contextual. In this paper we will re-examine the measurement process within a restriction of QM known as Stabilizer QM (SQM), that still exhibits for example Greenberger-Horne-Zeilinger nonlocality and Peres-Mermin contextuality. The re-examination will use a recent extension of SQM, the Contextual Ontological Model (COM), where the system state gives a complete description of future measurement outcomes reproducing the quantum predictions, including the mentioned phenomena. We will see that this gives a contextual Heisenberg microscope [1] whose back-action can be completely described within COM, and that the associated randomness originates in the initial state of the pointer system, exactly as in the original description of the Heisenberg microscope. The presence of contextuality, usually seen as prohibiting ontological models, suggests that the contextual Heisenberg microscope picture could perhaps be enabled in general QM.

Introduction—The first formulation of the uncertainty relation [2, 3] could be read as an epistemological statement, since it allows the possibility that an electron can possess both a well-defined position and momentum but there is a limit to what we can know about the electron. In that setting, the Heisenberg microscope would indeed output the well-defined position through a collision that induces a change in the well-defined momentum, see Fig. 1.

However, Heisenberg [3] writes in his conclusions that "such speculations seem to us, to say it explicitly, fruitless and senseless." Bohr fills in [4]: "Now the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected. Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation." Although there is no direct deductive path between a forced inclusion of the agency of observation and the non-existence of an independent reality of the observed phenomena, Bohr's continued argument over several years established the notion of *complementarity* which is today widely seen as a foundational principle of QM.

Further developments by Bell [5] and Kochen-Specker [6, 7] (KS) tell us that any possible independent reality must give nonlocal and contextual outcomes from the pre-existing values: future outcomes must sometimes depend on which measurements have been performed, even if the future measurements commute with the performed ones. This is usually taken as a strong argument against an independent reality of the observed phenomena, even supported by experiment [8–10].

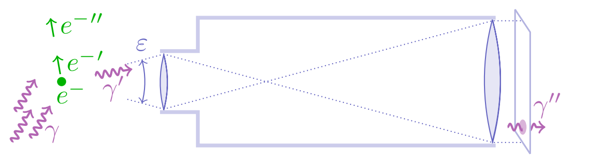

FIG. 1. The Heisenberg microscope. To measure the position of an electron e^- illuminate it with a pointer γ ray, that is scattered to γ' causing a recoil in the electron $e^{-\prime}$ and entangling the two. The scattered γ' position can then be measured through microscope optics, say creating a spot on a screen. The post-measurement state of $e^{-\prime\prime}, \gamma''$ is separable.

In quantum computing the role of position is played by the "computational basis" $|0\rangle$ and $|1\rangle$, the eigenstates of the Z operator (see Fig. 2), and momentum by the "phase basis" $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$, the eigenstates of the canonically conjugate X operator [11]. Even when restricting to only Pauli-tensor-product (Pauli-group) observables and preparations, so that one obtains Stabilizer QM (SQM) [12, 13], the theory still contains noncommuting observables and the uncertainty relation, and also displays Bell-nonlocality through the Greenberger-Horne-Zeilinger (GHZ) paradox [14] and KS-contextuality through the Peres-Mermin square [15, 16].

From the above argument, one could therefore expect that Bohr's conclusion also holds for SQM. However, SQM can be extended into the Contextual Ontological Model (COM) [17] that reproduces all predictions of SQM including Bell-nonlocality and KS-contextuality, even though all measurement outcomes are generated de-

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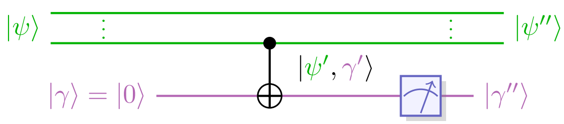

FIG. 2. The qubit Heisenberg microscope. To measure in the computational basis (qubit "position") on one of many qubits in the joint state $|\psi\rangle$ let the measured qubit interact with a pointer qubit $|\gamma\rangle = |0\rangle$, perhaps of a different type where you have access to a measurement device, through a CNOT (qubit "collision") to create the entangled $|\psi', \gamma'\rangle$, then measure on the pointer qubit. The post-measurement state $|\psi'', \gamma''\rangle$ is separable.

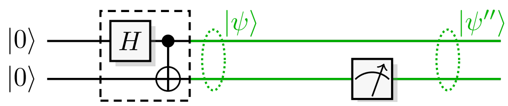

FIG. 3. Pauli-Z measurement on the last qubit of the entangled $|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$, stabilized by XX and ZZ.

terministically from the COM state. Thus *an independent reality in the ordinary physical sense can be ascribed to the phenomena* of SQM. This paper intends to show that this also applies to *the agencies of observation*, including the state update of a system measured through the Heisenberg microscope.

The argument in brief—SQM represents QM states not as complex vectors but indirectly as a set of Pauli-group observables that stabilize (preserve) the state. To obtain a unique stabilized state it is enough to write down a small set of commuting stabilizers (the *generators* of the *stabilizer group*), this is the motivation to use SQM. If the measured observable commutes with all generators, and then the outcome is fixed by the generators, otherwise the outcome is random ± 1 with equal probability. Calculating the outcome requires us to find which generators are contained in the observable.

This calculation is much simpler to do when using a helper set of *destabilizers* [18], one Pauli-group observable per generator that does not commute with that generator but commutes with all the others. Adding the helper set of destabilizers gives us the Stabilizer State Tableau Representation (SSTR) [18]. The SSTR state consists of a set of stabilizer generators paired with equally many destabilizers, for example

$$|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2} \simeq \{\underbrace{XX}_{\text{stab}}, \overbrace{ZZ}^{\text{stab}};\ \underbrace{Z\mathbb{I}}_{\text{destab}}, \overbrace{\mathbb{I}X}^{\text{destab}}\}. \tag{1}$$

The key insight of the Contextual Ontological Model (COM) [17] is that the set of stabilizers and destabilizers form a basis for the Pauli group, so *any* Pauli-group observable can be expressed in the elements, perhaps multiplied with additional ± 1 phases, here a and b in

$$\begin{aligned} |\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2} &\simeq \{XX, ZZ;\ aZ\mathbb{I}, b\mathbb{I}X\}, \\ &a, b = \pm 1 \text{ eq prob indep.} \end{aligned} \tag{2}$$

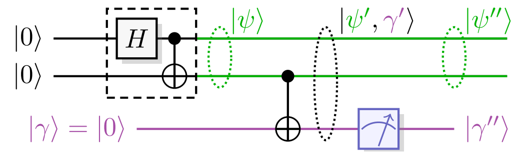

FIG. 4. The contextual Heisenberg microscope for a Pauli-Z measurement on the last qubit of the entangled $|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$, stabilized by XX and ZZ.

It is now possible to use COM to describe the qubit Heisenberg microscope. In particular it is possible to ask where and how the randomness enters, and we can follow "the finite and uncontrollable interaction between the objects and the measuring instruments" [19] in detail. In the example in Fig. 4, the measured system is in the COM state of Eqn. (2) and the pointer system is in the state

$$|\gamma\rangle = |0\rangle \simeq \{Z;\ cX\}, \quad c = \pm 1 \text{ eq prob.} \tag{3}$$

This makes the joint state

$$\begin{aligned} |\psi, \gamma\rangle &= (|000\rangle + |110\rangle)/\sqrt{2} \\ &\simeq \{XX\mathbb{I}, ZZ\mathbb{I}, \mathbb{II}Z;\ aZ\mathbb{II}, b\mathbb{I}X\mathbb{I}, c\mathbb{II}X\}, \\ &\qquad a, b, c = \pm 1 \text{ eq prob indep.} \end{aligned} \tag{4}$$

The next step is to entangle the pointer to the measured system through a CNOT. The usual CNOT map $|10\rangle \leftrightarrow |11\rangle$ and $|+-\rangle \leftrightarrow |--\rangle$ that preserves $|00\rangle$, $|01\rangle$, $|++\rangle$ and $|-+\rangle$, corresponds to a mapping between Pauli-group elements $X\mathbb{I} \leftrightarrow XX$ and $\mathbb{I}Z \leftrightarrow ZZ$ that preserves $\mathbb{I}X$ and $Z\mathbb{I}$, see details in [1]. Here, the CNOT creates the tripartite entangled GHZ state [14]

$$\begin{aligned} |\psi', \gamma'\rangle &= (|000\rangle + |111\rangle)/\sqrt{2} \\ &\simeq \{XXX, ZZ\mathbb{I}, \mathbb{I}ZZ;\ aZ\mathbb{II}, b\mathbb{I}XX, c\mathbb{II}X\} \end{aligned} \tag{5}$$

The measurement update of COM creates a post-measurement system state

$$\begin{aligned} |\psi'', \gamma''\rangle &= \begin{cases} |000\rangle \text{ if } a = +1, \\ |111\rangle \text{ if } a = -1 \end{cases} \\ &\simeq \{a\mathbb{II}Z, ZZ\mathbb{I}, \mathbb{I}ZZ;\ dXXX, bX\mathbb{II}, cXX\mathbb{I}\}, \\ &\qquad d = \pm 1 \text{ eq prob.} \end{aligned} \tag{6}$$

For the measurement in Fig. 3, COM the post-measurement state destabilizer has a random phase. Here, the destabilizer phase c originates from the unknown initial pointer-system destabilizer cX present in Eqn. (3). The state update of the measured system in the contextual Heisenberg microscope is not random, it is pre-determined by the pointer state, just as in the original Heisenberg microscope. A general proof that this conclusion always holds in the COM description of SQM can be found in [1].

Conclusions—The Contextual Ontological Model (COM) [17] is an EPR-complete [20] model for Stabilizer Quantum Mechanics (SQM) [12, 18, 21] that allows simultaneous well-defined pre-existing values associated with incompatible measurements. COM has a random state update at measurement that creates an upper limit to an observer's knowledge about future outcomes of these incompatible measurements, thus reproducing the uncertainty relation.

In this paper we have shown that within that model, we can restore the role of the Heisenberg microscope to the one first proposed by Heisenberg [2], noting that the microscope must be contextual to reproduce the SQM predictions. The contextual Heisenberg microscope performs a state update that is no longer random, but instead pre-determined by the initial state of the pointer system. There is no such mechanism in SQM itself and indeed, if one were to follow Bohr's argument, no such mechanism could exist in SQM. Nonetheless, the contextual Heisenberg microscope within COM realizes that mechanism, even though COM reproduces all predictions of SQM. This also shows that the traditional quantum measurement treatment involving a macroscopic measurement device or the environment is not cruical for tracing the origin of the random element causing the uncertainty. It suffices to consider a pointer system of the same type as the one measured upon, in our example a qubit system.

Lack of knowledge of the initial state of the pointer system would serve as an epistemic restriction on the initial COM state preparation, e.g., fixing the pointer computational basis state $|0\rangle$ but leaving a random initial phase of the COM state $\{Z; cX\}$. The completely deterministic state update of the contextual Heisenberg microscope will then cause the random initial pointer phase to randomize the existing predicted measurement outcomes of observables that do not commute with the measured observable. This forms an epistemic horizon from a deterministic law [22]; even though an EPR-complete model can describe the phenomena, an observer cannot obtain complete knowledge about the model state, because the initial state of the pointer is unknown to the observer.

Thus we can conclude that *any observation of SQM phenomena will involve an interaction with the agency of observation not to be neglected. But in spite of this an independent reality in the ordinary physical sense can be ascribed to the phenomena as well as to the agencies of observation.* COM reproduces all the SQM phenomena that are usually taken to forbid an ontological underlying model, including contextuality and nonlocality, and this suggests that the contextual Heisenberg microscope could be useful outside the restriction to SQM.

Acknowledgments—This work is supported by the Swedish Research Council project no 2023-05031.

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