Quantum Theory Can Decohere from a Causally-Indefinite Post-Quantum Theory: Extended Abstract arXiv:2511.02772

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We find a process satisfying the axioms of hyper-decoherence which produces standard quantum theory from the theory of quantum boxes (higher-order quantum theory with the non-signalling tensor product). This hyper-decoherence map evades the no-go theorem of Lee and Selby [8] by relaxing constraints on signalling to the past and the uniqueness of purifications. We discuss some natural opposing conclusions: that the existence of this map might be evidence of a genuine hyper-decoherence process producing causal quantum theory from its causally-indefinite higher-order theory; or that it serves as an indication that the axioms of hyper-decoherence might need careful re-consideration, especially regarding the subtle albeit central role that purity plays.

1 Introduction

One way to understand the emergence of classical theory from quantum theory is via the process of decoherence, in which the limited power or control of observers forbids them direct access to the quantum realm. In the knowledge that quantum theory is not a complete description of our physical world (for instance in its lack of a satisfactory accommodation of gravity), one is naturally led to the question of how quantum theory could emerge from some yet-to-be-defined post-quantum theory, via an analogous process of *hyper*-decoherence due to constraints on the powers of purely-quantum observers.

The search for such post-quantum theories appears to be cut short by the no-go theorem of [8], which establishes that there can be no such post-quantum theory satisfying both causality and the existence of unique purifications. Naturally, one can question the validity of these assumptions in the hope of bypassing this theorem, and a number of partial toy theories have been suggested which exhibit some notion of hyper-decoherence [9, 2, 4, 6], each one breaking, more or less severely, at least one axiom we would expect of a physically reasonable theory [7] or of a physically reasonable hyper-decoherence map, thereby allowing them to side-step the no-go theorem.

In the elusive search for a satisfactory post-quantum theory, a more principled approach could be to examine toy models of aspects of quantum gravity, which we expect to be a theory more fundamental than traditional quantum theory. Notably, an influential proposal of [5], that theories which unify quantum theory with gravity ought also to exhibit non-fixed causal structures, appears to draw a striking parallel with the non-causal conclusion of the no-go of [8]. This in turn begs the question,

Could quantum theory with indefinite causal order be that elusive post-quantum theory which hyper-decoheres to traditional quantum theory?

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2 Main Results

In the linked preprint we argue the affirmative, by finding a map satisfying the axioms of hyper-decoherence from the *theory of quantum boxes* QBox into standard quantum theory, where QBox is the most natural fragment of higher-order quantum theory for modelling indefinite causal order.

In order to bypass the no-go theorem [8] we allow for two relaxations of their axioms. Firstly, we relax the notion of causality that a theory must satisfy to permit a compatible family of possible ways of normalising systems, which we call a *multi-environment structure* similarly to [3]. Nevertheless, in this weakened setting it is still possible to define when processes are causal and no-backwards-signalling, generalising the standard definitions [1]. Multi-environment structures are introduced to allow for the non-uniqueness of causal effects in higher-order quantum theory.

The second relaxation is to allow the theory to possess non-unique purifications. In QBox, every process has a purification but these are not necessarily equivalent up to isometries on the purifying system.

With these alterations in place we proceed much like [8] to ask that our hyper-decoherence map `hypdec` satisfies the following axioms:

Ax1: it is idempotent so that once hyper-decoherence has occurred any further hyper-decoherence leaves the systems invariant,

Ax2: it is no-backwards-signalling, banning signalling from the future into the past,

Ax3: it copreserves the purity of states in $\mathscr{C}$, so that any state which hyper-decoheres to a pure state of $\mathscr{C}$ must be pure in $\mathscr{D}$,

Ax4: it preserves maximal mixtures, meaning that when hyper-decoherence is applied to a maximally mixed state of $\mathscr{D}$ it returns a maximally mixed state in $\mathscr{C}$.

The only notable change here is in Ax2 where we must ask that `hypdec` is no-backwards-signalling which takes the place of being normalised/causal.

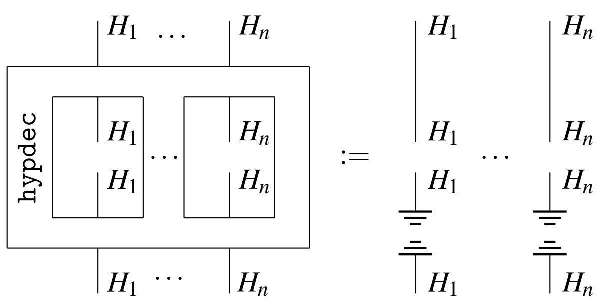
This allows us to make the following definition of what it means for one process theory to hyper-decohere to another.

Definition 1. A process theory $\mathscr{D}$ *supports hyper-decoherence* to a process theory $\mathscr{C}$ if every system H of $\mathscr{D}$ possesses a hyper-decoherence map $\texttt{hypdec} : H \to H$ satisfying axioms Ax1 - Ax4, such that the full sub-process theory spanned by hyperdecohered systems of the form is equivalent to $\mathscr{C}$, and at least one of the hyper-decoherence maps is not the identity process.

This allows us to define what it means for a theory to hyper-decohere to quantum theory.

Definition 2. A process theory $\mathscr{D}$ is *post-quantum* if it supports hyper-decoherence to the process theory CPTP.

We then find a family of maps in QBox,



and show that they satisfy the previous axioms, thereby proving the following theorem.

Theorem 1. QBox *is a post-quantum theory.*

These hyperdecoherence maps can be understood as implementing a transition from higher to lower order and they have a natural interpretation in terms of the reduction in power of observers by forbidding them access to one direction of the quantum boxes.

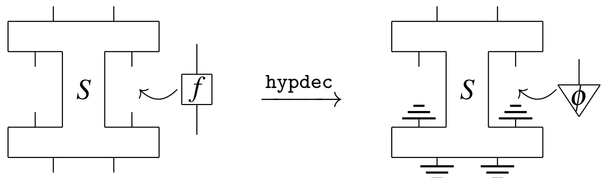


Figure 1: Hyper-decoherence of causal quantum theory from causally-indefinite higher-order quantum theory.

Our result gives the first example of a post-quantum theory exhibiting hyper-decoherence to quantum theory that does not suffer from some substantial foundational problem, and whose hyper-decoherence mechanism satisfies the most minimal of relaxations of the axioms of [8].

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