Simulating magic state cultivation with few Clifford terms (arXiv:2509.08658)

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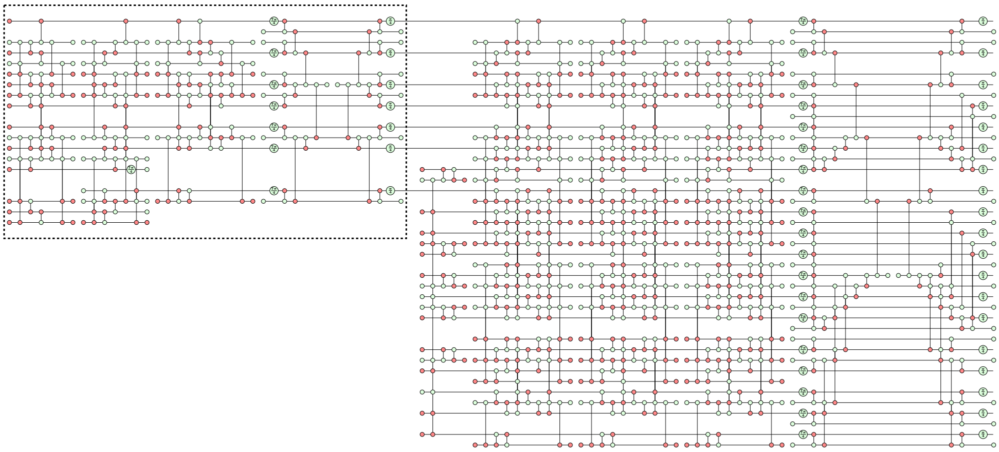

(1)

1 Introduction

Building upon [1], we present a few methods on how to simulate the non-Clifford $d = 5$ magic state cultivation circuits [2] with a sum of ≈ 8 Clifford ZX-diagrams on average, at 0.1% noise. Compared to a magic cat state stabiliser decomposition of all 53 non-Clifford spiders (6,377,292 terms required), this is more than 7×10^5 times reduction in the number of terms. Our stabiliser decomposition has the advantage of representing the final non-Clifford state (in light of circuit errors) as a sum of Clifford ZX-diagrams. This will be useful in simulating the escape stage of magic state cultivation, where one needs to port the resultant state of cultivation into a larger Clifford circuit with many more qubits. Still, it is necessary to only track ≈ 8 Clifford terms. Our result sheds light on the simulability of operationally relevant, high T-count quantum circuits with some internal structure.

We then provide numerical results for full non-Clifford stabiliser rank simulation based on `tsim` along with optimisations using our cutting decompositions. Nearly 4×10^6 shots per second can be obtained on a laptop for the smaller $d = 3$ circuits at SD6 circuit-level noise $p = 0.0005$, making it only ~ 1.1 times slower than its (circuit-unspecific and un-optimised) fully Clifford proxy simulation via `stim` using S gates.

2 Background and motivation

The $d = 5$ magic state cultivation circuit [2], shown in (1), is difficult to simulate due to its high T-count of 53 $T(T^\dagger)$ gates. The authors of [2] simulated most of the larger $d = 5$ circuits by replacing $T(T^\dagger)$ gates entirely with $S(S^\dagger)$ gates. Applying the same replacement to the $d = 3$ sub-circuit (inside the black dashed box of (1)) reveals an $\approx 2\times$ discrepancy in logical error rate (LER) when benchmarked against brute-force state vector simulation. This demonstrates the need for simulation techniques that handle the non-Clifford gates exactly while remaining compatible with Monte Carlo error sampling.

3 Method and decomposition

In this work, we use techniques from [1] to decompose the $d = 5$ magic state cultivation circuit from (1) into $\sim$ 8 Clifford ZX-diagrams on average, while fully accounting for Pauli errors on every edge in the ZX-diagram at error rates in the operationally relevant regime of 10^{-4} to 10^{-3}. A meaningful benchmark of logical performance[1] without the $T(T^\dagger) \to S(S^\dagger)$ substitution may therefore be feasible, given sufficient integration of ZX-calculus-based stabiliser decomposition into tools such as `tsim` [3].

For the smaller $d = 3$ sub-circuit, we complement the analytical decomposition with a full numerical simulation pipeline. By combining spider cutting with BSS stabiliser decomposition, the circuit compiles into 120 Clifford graphs with a total of 28 node terms, 192 sign terms, and 1,408 phase-pair terms. We implement several optimisations on top of `tsim` [3]: BLAS-accelerated evaluation via matmul-based binary row sums, enumeration-based sampling over the 32 measurement-outcome combinations, noiseless outcome caching, and full-program JIT compilation.

We further accelerate sampling by replacing per-shot channel sampling with a sparse geometric-skip sampler that generates events only at non-identity fire positions, deduplicating repeated noise-channel patterns with a persistent cross-batch evaluation cache, and compiling the combo-evaluation loop via `jax.lax.scan`. Together, these yield a throughput exceeding 4 million shots per second on a laptop (Apple M4 Macbook Pro) at circuit-level noise $p = 5 \times 10^{-4}$, only $\sim 1.1\times$ slower than a fully Clifford proxy simulation via `stim`, partially addressing the open problem of simulating magic state cultivation on common day-to-day hardware posed by [2]. Note that our pipeline is optimised specifically for this circuit, while `stim` is a general-purpose Clifford simulator. This confirms a logical error rate of $\sim 10^{-6}$ at $p = 10^{-3}$ for the $d = 3$ circuit without resorting to the $T \to S$ substitution or the slower statevector simulations.

4 Results

We note that several concurrent works on magic state cultivation have recently appeared [4–6] or been updated [7] on the arXiv. It would be interesting to investigate whether our simulation method extends to other cultivation circuit variants [8]. We note that the throughput in our numerical experiments is several orders of magnitude higher than that of the competing approach [6] and outperforms [9] in terms of average shots per second by a factor of 11.7 (at $p = 10^{-3}$), without the aid of a pricey H800 Nvidia GPU[2]. Extending the full numerical pipeline to the $d = 5$ cultivation circuit, potentially with GPU-accelerated `tsim`, is left for future work.

[1] And any extension to simulate the escape stage end-to-end.

[2] Googled price of Nvidia H800 as of Feb 2026, ~\$30,000 (USD). The Macbook Pro used for our computation cost ~\$2,000 (USD).

Table 1: Sampling throughput for the $d = 3$ cultivation circuit ($N = 5$, $M = 32$, $K = 2$, batch size 65,536) on Apple M4 Pro series CPU at $p = 0.001$ with 2^{26} shots. All methods are exact.

Optimisation	shots/s	Speedup	Key change
Autoregressive baseline	∼20k	1.0×	Sequential $N+1$ evaluation levels
+ Enumeration + BLAS matmul	∼38k	1.9×	Single-level eval of all $M=32$ combos
+ Complex64 LUT	∼42k	2.1×	Float32 lookup replaces exact integer arith.
+ Full-program JIT + noiseless cache	∼54k	2.7×	No Python overhead; cache ∼64% noiseless shots
+ Split f/m decomposition	∼94k	4.7×	32× matmul reduction via row-sum linearity

Table 2: Accelerated sampling optimisations for the $d = 3$ cultivation circuit ($N=5$, $M=32$, $K=2$, batch size 2,097,152) on Apple M4 Pro CPU at $p = 10^{-3}$ with 2^{29} shots. Speed-ups are relative to the split f/m baseline of table 1. All methods are exact.

Optimisation	shots/s	Speedup	Key change
Split f/m baseline (table 1)	∼94k	1.0×	Batch size 65,536
+ Inverse CDF + deduplication	∼974k	10.4×	$\bar{K}\times$ fewer draws; eval only $U \ll B$ unique f-patterns
+ Geometric skip + persistent cache + scan	∼2,778k	∼30×	Sparse fire events; cross-batch cache; single-body scan

Table 3: Comparison of simulators for the $d = 3$ magic-state cultivation circuit at $p = 10^{-3}$.

	This work [10, 11]	**SOFT** [9]	**Surti et al.** [6]
Method	ZX cutting [12] + `tsim` [3]	Extended stabiliser tableau [13]	Clifford expansion
Hardware	Apple M4 Pro (CPU)	Nvidia H800 (GPU)	Not reported
Shots/s	2,777,295	∼237,000	∼100
Exact (non-Clifford)	✓	✓	✓
Hardware cost	∼$2,000 (common computer)	∼$30,000	Not reported

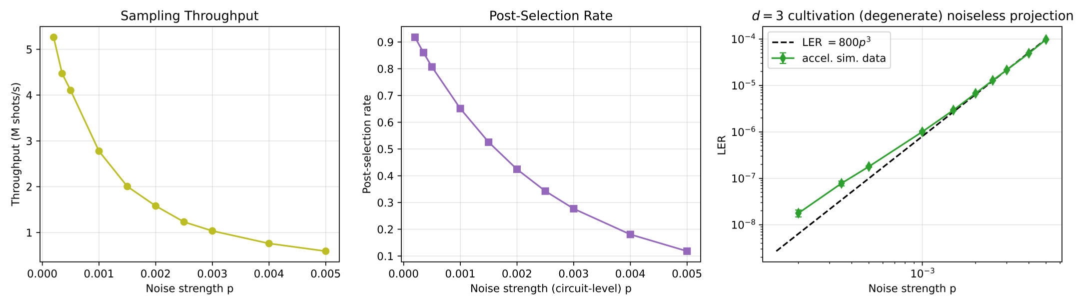

Figure 1: Accelerated exact T gate simulation results for the $d = 3$ cultivation circuit (degenerate injection, noiseless projection) on Apple M4 Pro CPU. **Left:** Sampling throughput, reaching ~ 4.1 million shots/s at $p = 5 \times 10^{-4}$. For reference, the S gate Clifford proxy via `stim` achieves ~ 4.5 million shots/s, making our exact pipeline only $\sim 1.1\times$ slower. **Centre:** Post-selection rate, decreasing from ~ 0.92 at $p = 5 \times 10^{-4}$ to ~ 0.12 at $p = 5 \times 10^{-3}$. **Right:** Logical error rate versus noise strength; the data (green) are consistent with $\mathrm{LER} = 800\,p^3$ (dashed), confirming the distance degrading effect as seen in [2] for decreasing p.

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