Reference frames for process matrices: from coordinate parametrization to spacetime representation

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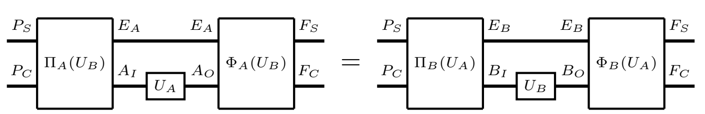
In recent years, two prominent research lines have developed complementary languages for describing spacetime and causality in quantum theory: quantum reference frames (QRFs) [1–17] and the process-matrix formalism [18–22]. Although developed in parallel, they meet on common ground: both accommodate situations in which the causal relation between events is indefinite. A key conceptual distinction for the analysis below is that "coordinates" and "reference frames" can be understood in two distinct, yet standard, ways. On the one hand, coordinates may be treated as *abstract labels*: one introduces a parametrization (times, positions, slots in a circuit) and requires that the empirical content of the theory, i.e. observable predictions, does not depend on the chosen labels. On the other hand, coordinates may be understood as *physical instantiations*: they arise from physical systems (fields, rods, clocks) whose readings define the coordinate assignment. Spatiotemporal QRFs [9, 10, 12, 13, 16, 17, 23–25] push both viewpoints into the quantum regime: quantum coordinates and quantum reference fields serve as "quantum rods" and "quantum clocks" and provide a relational coordinatization of the remaining degrees of freedom, naturally accommodating superpositions of event locations and, in gravitational settings, superpositions of causal relations. In both viewpoints, however, a central question remains: *how does one transform between different descriptions?* In quantum theory, changes of description that preserve physical content are typically represented by unitary (or, more generally, isometric) maps. This is the operative meaning of "same physics in different coordinates/frames": probabilities, operational predictions, and the structure of admissible transformations must be preserved across viewpoints. The process-matrix framework characterizes the full space of correlations compatible with quantum mechanics at each local laboratory without assuming a fixed global causal order. A process matrix is a higher-order quantum map [26–30] sending local quantum operations to probabilities. It encompasses causally separable processes, causally nonseparable processes such as the quantum switch [31, 32], and also processes that violate causal inequalities [18, 20, 22, 33–35], often called noncausal. A core open issue is *physical realizability*: which abstract processes can arise from bona fide dynamics of physical systems embedded in time–spacetime?

Causal reference frames and time delocalized subsystems decompositions. Two influential viewpoints sharpen the operational meaning of "agent perspective" within the process formalism. Causal reference frames (CRFs) [36] provide event-dependent parametrizations in which a chosen party appears local, while time-delocalized subsystems (TDSs) [33, 37–39] reinterpret indefinite causal order as time delocalization of subsystems relative to an agent's time frame. Let us consider a unitary process W, where we identify with P_S, P_C and F_S, F_C the past and future target (S) and control (C) systems, while (A_I, A_O) and (B_I, B_O) are the input-output systems of Alice and Bob's laboratory respectively. Choosing a causal reference frame—or equivalently a TDS—amounts to decomposing the evolution of the full process into the past and future of a given laboratory, so that the corresponding agent is effectively local within its own time frame:

$$\langle\langle U_A|\langle\langle U_B||W\rangle\rangle =$$



(1)

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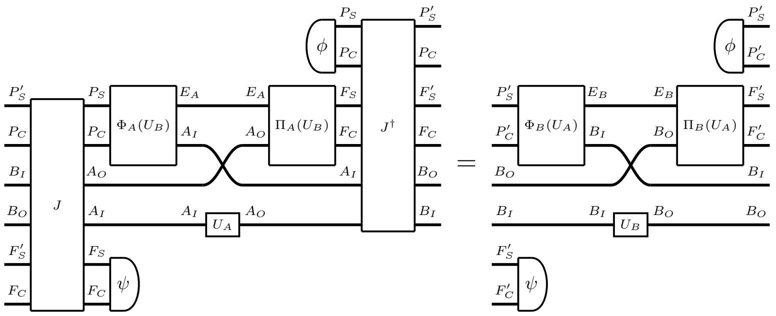
where $\Pi_{A/B}$, $\Phi_{A/B}$ are "future" and "past" functions sending unitaries to unitaries, which describe how the global process decomposes from the perspective of Alice's or Bob's causal frame. For pure processes, these representations are equivalent to the underlying process matrix in the precise sense that they yield the same higher-order map, i.e. the same operational statistics for all local instruments.

A no-go for slot preserving unitaries. At this point, however, a tension emerges that motivates the present work. On the one hand, if CRF/TDS representations are "just different descriptions" of the same physical process, one expects that moving between them should be implementable by a unitary change of perspective. On the other hand, it has been shown for the quantum switch that one cannot unitarily map the circuit fragments associated with one agent-local representation to the complementary one while keeping fixed the boundary-time identification (the "past" preparation and "future" measurement) [37, 39]. In other words, there is an operational equivalence of descriptions, but no unitary mapping of the corresponding fragmentations under the slot-preserving constraints: there exist no maps J and its conjugate such that



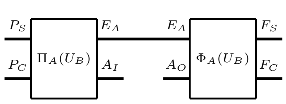
(2)

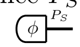
How can "same physical content" coexist with the apparent absence of a unitary change of perspective? *Time frame and the corresponding perspective-change transformation for processes.* We resolve this tension by taking seriously the dual role of coordinates. Our first claim is that CRF and TDS representations are most naturally understood as *coordinate parametrizations of a perspective-neutral process*, so their equivalence is a statement of coordinate invariance at the level of the *unfragmented* process.

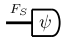
(1a) Coordinate parametrization.

(1b) Operational cut as time frame.

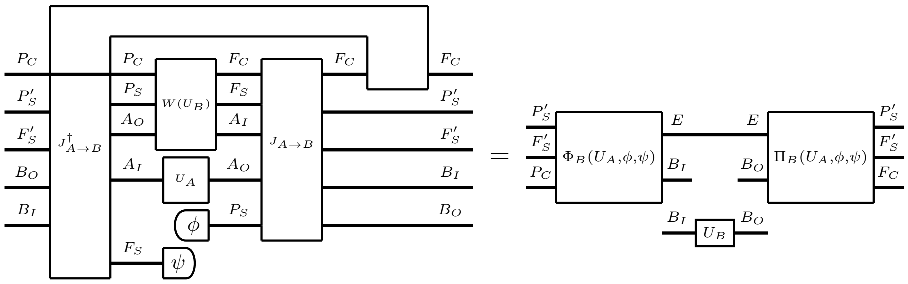
Graphically it amounts to 1a where t_0, t_1, t_2, t_3 are just bookkeeping of the circuit blocks—i.e. events. Our second claim is that a *physical perspective* enters only when one promotes the coordinate assignment to frame data—most notably, when the "time coordinate" is treated as *what a physical clock shows.* Operationally, this corresponds to an *operational cut* of the process into circuit fragments (events), thereby endowing the description with a definite time foliation relative to a chosen frame. In the resulting picture 1b, the symbols t_0, t_1, t_2, t_3 now label the *interfaces* between fragments and thus encode not only a parametrization but a *specific foliation* (boundary identification) of the process. Within this foliation, Alice's laboratory is local in the step (t_1, t_2), while Bob's intervention is in a superposition of temporal locations, i.e. delocalized in (t_0, t_1) and (t_2, t_3). Once such frame data are included, a correct change of perspective must act not only on the system degrees of freedom but also on the time-frame information itself. The no-go results then acquire a sharp meaning: they exclude unitary maps that *preserve the original slot identities* (i.e. keep the original time labels and boundary partition fixed), and therefore do not implement a genuine change of time frame. In Alice's foliation, Bob's gate appears in a coherent superposition of two temporal placements:

$|W(U_B)\rangle\rangle =$  $= |00\rangle\rangle^{F_C P_C}|I\rangle\rangle^{A_I P_S}|U_B\rangle\rangle^{F_S A_O} + |11\rangle\rangle^{F_C P_C}|I\rangle\rangle^{F_S A_O}|U_B\rangle\rangle^{A_I P_S}.$ (3)

one in the "global past" segment (t_0, t_1) with input–output systems (P_S, A_I), and one in the "global future" segment (t_2, t_3) with input–output systems (A_O, F_S). By contrast, Alice's own operation $|U_A\rangle\rangle^{A_O A_I} = \overset{A_I}{—}\boxed{U_A}\overset{A_O}{—}$ is local in her foliation, confined to the interval (t_1, t_2). Our goal is to pass to a foliation in which Bob's gate is local, but this cannot be achieved by any *slot-preserving* transformation, i.e. any unitary that keeps Alice's interface identities (and hence her past/future partition) fixed while attempting to reassign locality/delocalization. Indeed, Eq. (3) shows that "localizing" Bob necessarily requires a reassignment of the delocalized boundary subsystems P_S and F_S. In particular, since P_S and F_S sit at the original boundaries, any such reassignment must also transform the associated preparation 

and final measurement 

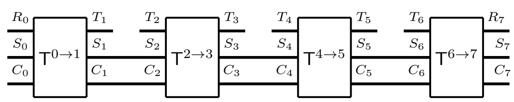
. The key ingredient is therefore to implement a *global* reshuffling of the foliation data:



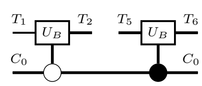
(4)

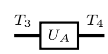
The transformed fragments of the preparation, the final measurement, and Alice's gate—identified by $\widetilde{W}$—do *not* coincide with those appearing in Bob's CRF under the *same* boundary identification: the degrees of freedom associated with the original preparation/measurement straddle the new past–future partition and thus become time-delocalized.

Spacetime frame for processes. The bare switch process contains only the degrees of freedom associated with the laboratories' input–output systems. It therefore provides no additional subsystem that could serve as an *external spatiotemporal background*—no clock/rod register relative to which all events could be localized. For this lack of frame resources, a unitary time-frame change that localizes Bob's operation unavoidably alters the global notion of "past" and "future". Then we supplied precisely this missing structure by extending the process with explicit quantum reference systems that carry accessible spatiotemporal information. These additional reference systems provide a *background circuit fragment*, i.e. a spatiotemporal scaffold into which the process embeds [40–42]:

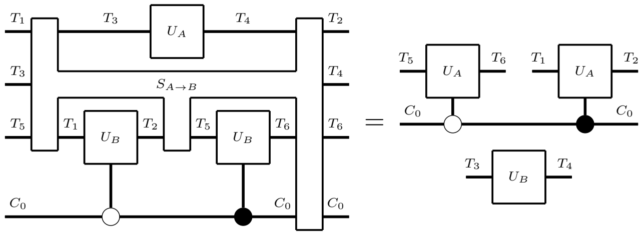
$$|W_{B|A}\rangle\rangle =$$


. (5)

While Bob's spacetime delocalized operation is 

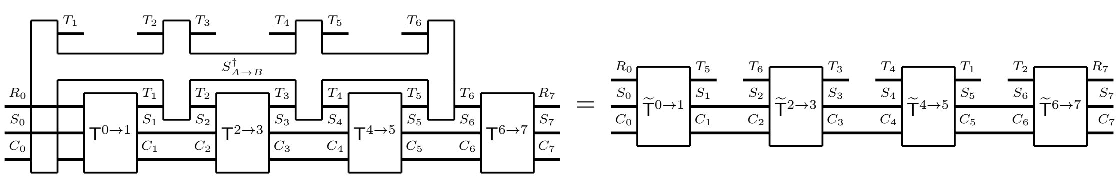
and Alice's localized one is 

. In this extended setting, we construct a unitary perspective-change transformation—$S_{A\to B}$—that maps the CRF/TDS fragments associated with one spacetime viewpoint (Alice) to the complementary ones (Bob), togheter with a different spatiotemporal frame, while preserving the global past and future. Hence, the transformed operations are



, (6)

which clearly coincide with Bob CRF/TDS. Whereas the transformed spacetime scaffold is



. (7)

Hence we transformed unitarily the fragments—gates and background—associated with Alice's CRF into the those of Bob's CRF. This provides a concrete route toward empirical realizability of process matrices through bona fide spacetime-and-matter dynamics. Furthermore, extending the same perspective-based construction beyond switch-like processes—e.g. unitary noncausal processes would test whether noncausality can be re-expressed as quantum spatiotemporal delocalization of laboratories within an acyclic spacetime, without invoking closed timelike curves.

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We study how to define and transform *frame perspectives* for quantum processes in the process-matrix formalism. We argue that, for pure processes, the causal reference frames (CRFs) and time-delocalized subsystems (TDSs) formalisms should be understood primarily as *coordinate parametrizations* of a single perspective-neutral higher-order object, rather than as descriptions tied to actual physical frames. A genuine time (or causal) *perspective* arises only once one endows the process with additional *frame data* by choosing an operational foliation into circuit fragments (events). With this distinction, existing no-go results acquire a clear scope: they rule out *slot-preserving* unitaries, i.e. transformations that attempt to switch perspectives while keeping the fragment boundaries—and hence the induced global past/future partition—fixed. Focusing on the quantum switch, we construct explicit perspective-change maps that do implement a unitary change of foliation, but necessarily reshuffle the boundary identification of past and future. We then show how to restore a unitary complementarity between perspectives by extending the process with explicit quantum reference systems that provide a shared spatiotemporal scaffold. In the extended setting, complementary CRF/TDS viewpoints become unitarily related while preserving a common global past and future. We discuss how this perspective-based approach informs the broader question of physical realizability of abstract process matrices.

I. INTRODUCTION

In recent years, two prominent research lines have developed complementary frameworks for describing spacetime and causality in quantum theory: quantum reference frames (QRFs) [1–23] and the process-matrix formalism [24–28]. Although developed in parallel, they meet on common ground: both accommodate situations in which the causal relation between events can be indefinite.

A key conceptual distinction—relevant for the analysis of both QRFs and the process-matrix formalism—is that "coordinates" and "reference frames" can be understood in two distinct, yet standard, ways. On the one hand, coordinates may be treated as *abstract labels*: one introduces a parametrization (times, positions, slots in a circuit) and requires that the empirical content of the theory, i.e. observable predictions, are independent on the chosen labels. On the other hand, coordinates may be understood as *physical instantiations*: they arise from physical systems (fields, rods, clocks) whose readings define the coordinate assignment. In this second sense, the coordinate fields themselves are dynamical, can interact with the systems they coordinate, and backreaction is in principle unavoidable. In general relativity (GR), both viewpoints are routinely employed: we use coordinate transformations (diffeomorphisms) to express the laws in different charts, and we also compare descriptions tied to different physical frames built from matter fields. The physical content of the theory is diffeomorphism-invariant, i.e., it is unchanged under coordinate changes on the spacetime manifold. Spatiotemporal QRFs [9, 10, 12, 13, 16, 17, 29–31] push both viewpoints into the quantum regime: quantum coordinates and quantum reference fields can serve either as abstract or as physical "quantum rods"

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and "quantum clocks," thereby providing a relational description of the remaining degrees of freedom. This framework naturally accommodates the possibility of superpositions of event locations and, consequently, superpositions of causal relations, both in gravitational and non-gravitational settings.

On both viewpoints, however, a central question remains: How does one transform between different descriptions, i.e., between descriptions associated with different QRFs or coordinate systems? In quantum theory, the changes of description that preserve physical content are typically represented by unitary (or, more generally, isometric) maps. This is the operational meaning of the expression "same physics in different coordinates/frames": probabilities, operational predictions, and the structure of admissible transformations must be preserved across perspectives.

The process-matrix framework characterizes the full space of correlations compatible with quantum mechanics at each local laboratory without assuming a global causal order. A process matrix is a higher-order quantum map [32–36] encompassing causally separable processes, causally nonseparable processes such as the quantum switch [37, 38], and also processes that violate causal inequalities [24, 26, 28, 39–41], often called noncausal. A core open issue is *physical realizability*: which abstract processes can arise from bona fide dynamics of physical systems embedded in spacetime? In Ref. [42] a background-independent formulation of the process-matrix framework was proposed, based on permutation invariance of the laboratory labels, and it was shown that under this requirement one loses the notion of distinct local operations unless a reference frame is used to distinguish them; in this paper, we explicitly retain the identity of the laboratories.

So far, two frameworks explored the operational meaning of "agent perspective" within the process-matrix formalism. Causal reference frames (CRFs) [43] provide event-dependent parametrizations in which a chosen agent's operation appears local, while time-delocalized subsystems (TDSs) [39, 44–46] reinterpret indefinite causal order as time delocalization of subsystems relative to an agent's time frame. For pure processes, these representations are equivalent to the underlying process matrix in the precise sense that they yield the same higher-order map, i.e. the same operational statistics for all local instruments.

At this point, a tension emerges. On the one hand, if CRF/TDS representations are "just different descriptions" of the same physical process, one expects that moving between them should be implementable as a symmetry transformation—hence, in the quantum setting, by a unitary change of perspective. On the other hand, it has been shown for the quantum switch that natural candidates for such transformations fail: one cannot unitarily map the circuit fragments associated with one agent-local representation to the complementary one of the other agent, while keeping fixed the time-frame data, i.e. the "past" preparation and "future" measurement [44, 46]. In other words, the two descriptions are operationally equivalent, yet there is no unitary transformation that maps one chosen circuit decomposition to the other while keeping fixed the identification of the preparation and measurement boundaries. How can "same physical content" coexist with the apparent absence of a unitary change of perspective?

We resolve this tension by taking seriously the dual role of coordinates. Our first claim is that CRF and TDS representations are naturally understood as *coordinate parametrizations of a perspective-neutral process*: they are rewritings of the same higher-order object, and their equivalence is a statement of coordinate invariance at the level of the *unfragmented* process. By a *fragment* we mean a time-ordered piece of the circuit, delimited by chosen interface boundaries. Our second claim is that a *physical perspective* enters only when one promotes the coordinate assignment to frame data—most notably, when the "time coordinate" is treated as *what a physical clock shows*. Operationally, this corresponds to an *operational cut* of the process into circuit fragments (events), thereby endowing the description with a definite time foliation relative to a chosen frame. Once such frame data are included, a correct change of perspective must act, not only on the system degrees of freedom, but also on frame data itself. The no-go result then acquires a sharp meaning: it excludes unitary maps that *preserve the original slot identities* (i.e. keep the original time labels and boundary partition fixed), and therefore does not implement a genuine change of time frame.

With this distinction in place, we proceed as follows. We formalize an agent-dependent time perspective via the operational cut of a CRF/TDS representation and construct explicit perspective-change maps. We show that a unitary change of perspective exists that takes a description in which one agent's operation is local to one in which the other agent's operation is local [47][48]; however, this unitary necessarily *reshuffles* the boundary notion of past and future, so that the two descriptions do not share a common global past and future. We then extend the process by introducing additional quantum reference systems that provide a background circuit fragment—i.e. a spatiotemporal scaffold into which the process is embedded [13, 17]. In this extended setting, we construct a unitary perspective-change transformation that maps the CRF/TDS associated with one spacetime viewpoint to the complementary CRF/TDS associated with a different spatiotemporal frame, while *preserving* a common global past and future.

This viewpoint does *not* require abandoning abstract coordinates. Rather, abstract coordinates can be understood as *gedanken coordinate fields*: fields that could, in principle, be instantiated as rods and clocks, but need not be. As in classical GR, one may express dynamical laws in arbitrary coordinates and prove coordinate invariance of physical statements. However, when one moves between coordinate choices interpreted as different counterfactual physical frames, the transformation must also act on the frame data. This is precisely where the quantum setting differs from the classical one: the "frame" may itself be quantum and entangled with the system, so the perspective change must account for that quantum nature.

The remainder of the paper is organized as follows. In Sec. II we recall the CRF and TDS formalisms and recast their equivalence as coordinate invariance of the perspective-neutral process. In Sec. III.1 we introduce an agent-dependent time perspective via an operational cut and construct explicit time-frame-change transformations, clarifying the scope of existing no-go results. Finally, in Sec. IV we extend the quantum switch with reference systems providing a spatiotemporal scaffold and show how complementary spatiotemporal perspectives become unitarily related in the extended description.

II. CAUSAL REFERENCE FRAMES AND TIME-DELOCALIZED SUBSYSTEMS REPRESENTATIONS

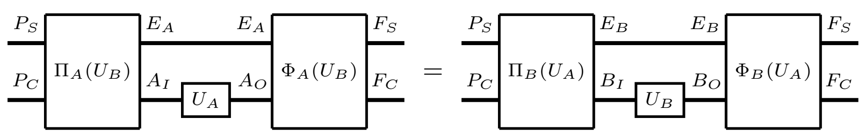
In Ref. [43] it was shown that any pure bipartite process matrix, with global past and future can always be expressed in equivalent decompositions called causal reference frames (CRFs). Let us consider a unitary process W, where we identify with P_S, P_C and F_S, F_C the past and future target (S) and control (C) systems, while (A_I, A_O) and (B_I, B_O) are the input-output systems of Alice and Bob's laboratory respectively. Choosing a causal reference frame amounts to decomposing the evolution of the full process into the past and future of a given laboratory, so that the corresponding agent is effectively local within its own time frame. In this setting, the equivalence between the causal reference frames decomposition representation of the process is expressed as

$$|W\rangle\!\rangle = \sum_i \langle\!\langle I|^{E_A E_A} |\Pi_A(U_i)\rangle\!\rangle^{E_A A_I P_S P_C} |\Phi_A(U_i)\rangle\!\rangle^{F_S F_C E_A A_O} |U_i\rangle\!\rangle^{B_O B_I} \tag{1}$$

$$= \sum_i \langle\!\langle I|^{E_B E_B} |\Pi_B(V_i)\rangle\!\rangle^{E_B B_I P_S P_C} |\Phi_B(V_i)\rangle\!\rangle^{F_S F_C E_B B_O} |V_i\rangle\!\rangle^{A_O A_I}, \tag{2}$$

where $E_{A/B}$ are suitable isomorphic ancillary systems, whereas $\{U_i\}$ and $\{V_i\}$ are two orthonormal unitary basis for B and A respectively. Here, $\Pi_{A/B}$, $\Phi_{A/B}$ are "future" and "past" functions mapping unitaries to unitaries, which describe how the global process decomposes from the perspective of Alice's or Bob's causal frame. Hence for any choice of unitary operations U_A of Alice and U_B of Bob we have

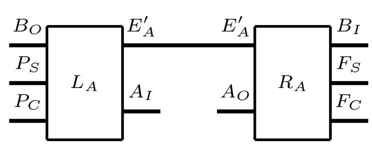
$$\langle\!\langle U_A|\langle\!\langle U_B||W\rangle\!\rangle = |W(U_A, U_B)\rangle\!\rangle =$$



. (3)

In Ref. [43] the authors showed the link between CRFs and Oreshkov's time-delocalized subsystems decomposition [44]. Specifically, the same unitary process in the time-delocalized decomposition is $|W\rangle\!\rangle = \langle\!\langle I|^{EE}|L_A\rangle\!\rangle^{EA_IPB_O}|R_A\rangle\!\rangle^{FB_IEA_O}$, which expressed as a quantum circuit

$$|W\rangle\!\rangle =$$



, (4)

makes evident the time delocalization of Bob's output and input in the global past and future of Alice's localized operation, respectively. Note that $E'_{A/B}$ are ancillary systems isomorphic to $E_{A/B}$, while the subscripts in L_A and R_A merely indicate that the process is being expressed in Alice's time frame. At this stage no agent operation has been applied, hence L_A and R_A do not depend on U_A or U_B.

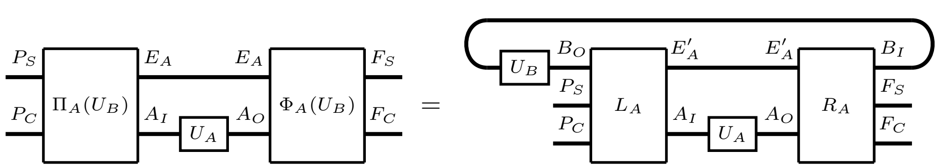
Hence, the two decompositions are related as follows

$$|W(U_A, U_B)\rangle\!\rangle = \langle\!\langle I|^{E_A E_A} \langle\!\langle U_A|^{A_O A_I} |\Phi_A(U_B)\rangle\!\rangle^{E_A A_I P_S P_C} |\Pi_A(U_B)\rangle\!\rangle^{F_S F_C E_A A_O} \tag{5}$$

$$= \langle\!\langle U_B|^{B_O B_I} \langle\!\langle U_A|^{A_O A_I} \langle\!\langle I|^{E'_A E'_A} |L_A\rangle\!\rangle^{E'_A A_I P_S P_C B_O} |R_A\rangle\!\rangle^{F_S F_C B_I E'_A A_O}, \tag{6}$$

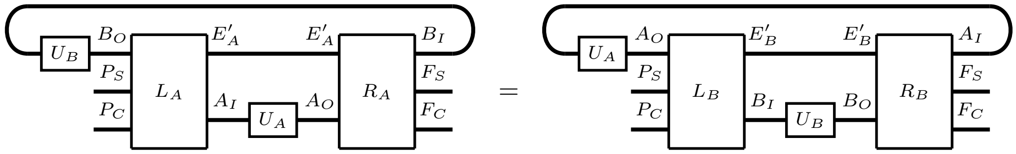
The latter equation is graphically displayed by

$$|W(U_A, U_B)\rangle\!\rangle =$$



. (7)

Therefore, the same equivalence in Eq. (3) can be expressed in terms of the time-delocalized decomposition of W, namely



(8)

where L_A, R_A and L_B, R_B are simply related by a change of subsystem factorization [46].

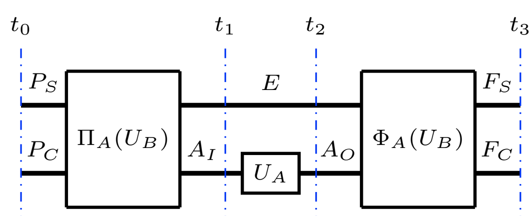
In what follows, we recast the equivalences in Eqs. (3) and (8) as instances of coordinate invariance. More specifically we will point out that different CRFs or TDSs decompositions are rather alternative parametrizations of the abstract process, than genuine physical perspective provided by a choice of reference frame.

III. NO-GO FOR UNITARIES PRESERVING THE TIME FRAGMENTATION

III.1. Coordinate time parametrization and foliation into fragments

For the purposes of this paper it is useful to keep two levels of description distinct. At the level of the *perspective-neutral* (higher-order) process there is no preferred global time parameter: any "time labels" attached to boxes or wires merely parametrize a chosen representation, and the operational content is invariant under such reparametrizations. A time perspective enters only once one chooses an explicit *foliation* (fragmentation) into circuit fragments that are interpreted as "events" in an agent's description. The difference will be illustrated by the two figures below.

Coordinate parametrization. Let us start with the circuit in Alice's causal reference frame (equivalently, a TDS decomposition) viewed purely as a *coordinate parametrization* of the same underlying process. We introduce labels t_1 and t_2 to mark the beginning and the end of Alice's local gate, and we denote the global input and output boundaries by t_0 and t_3, respectively. In this parametrization Bob's gate appears time-delocalized, as illustrated below:

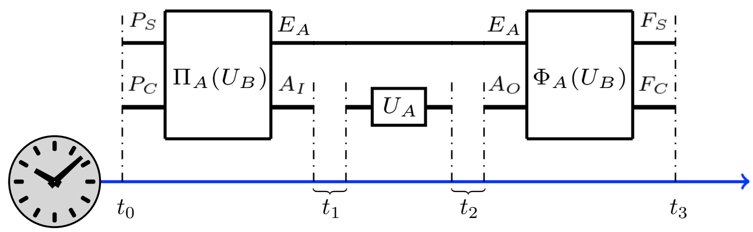


(9)

We stress that the "time delocalization" displayed here is *not* defined relative to any physical clock or time-reference system. Rather, it is an intrinsically *relational* notion that appears only after choosing a causal reference frame (or, equivalently, a TDS decomposition): one party is represented as local by construction, and the other is then represented as delocalized *relative* to that choice. Consequently, for a quantum-switch–like process there is, at this stage, no intrinsic temporal localization with respect to a third system that could serve as a background clock. The symbols t_0, t_1, t_2, t_3 in Eq. (9) therefore function only as *coordinate parameters*—bookkeeping devices for a particular representation of the same underlying higher-order object. They should not be conflated with a physical perspective, where "time" is literally the readout of a clock degree of freedom, relative to which events are defined and which is therefore, in principle, subject to backreaction and quantum correlations.

Foliation into fragments To pass from a mere parametrization to an agent-dependent *time perspective* one must choose a concrete foliation of the process into circuit fragments that are to be regarded as "events" in the description. Operationally, this corresponds to introducing an *operational cut*: we cut the target wires in the vicinity of the labels t_1 and independently t_2, thereby decomposing the process into Alice's local gate and Bob's time-delocalized operation. In the resulting picture, the same symbols t_0, t_1, t_2, t_3 now label the *interfaces* between fragments and thus encode not only a parametrization but a *specific foliation* (boundary identification) of the process. Within this foliation, Alice's laboratory is local in the step (t_1, t_2), while Bob's intervention is in a superposition of temporal locations, i.e.

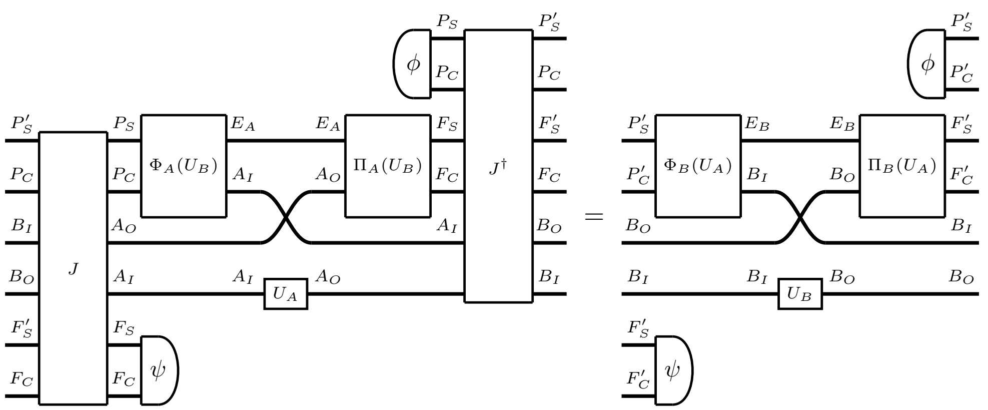
delocalized in (t_0, t_1) and (t_2, t_3) as illustrated in the following figure:



(10)

Even here, however, the cut by itself does *not* yet imply that t_i are readings of a *physical* clock: absent additional reference information, the interfaces are still defined only relative to the chosen decomposition. What has changed is that the description now carries *extra structure* beyond the timeless process matrix, namely a distinguished foliation into fragments—a choice of "event boundaries"—where one may insert extra systems, gates, or implement transformations by acting on the corresponding boundary input–output systems. A *genuine* physical time frame—"t_i as what a clock shows"—requires further structure, e.g. an explicit reference system whose distinguishable states are correlated with these interfaces, making the boundary data operationally accessible as frame information. This is precisely the role of the background scaffold introduced later in the paper.

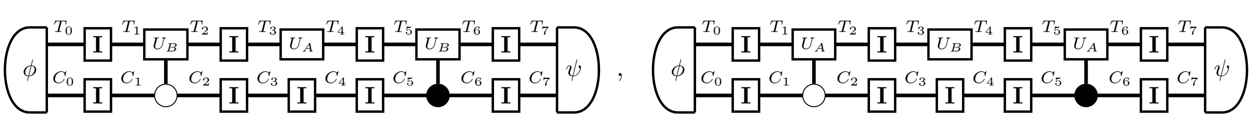
A natural question is whether the equivalence in Eqs. (3) and (8) can be realized at the level of *fragmented* descriptions, i.e. whether one can unitarily "re-frame" the fragments obtained from Alice's foliation so that they coincide with those obtained by cutting the process in Bob's CRF. In Refs. [44, 46] it was shown that such a mapping is impossible for a specific class of unitaries: there exist no maps J (and its conjugate) acting at the boundary interface times t_0 and t_3, as shown in Fig. (10) such that



, (11)

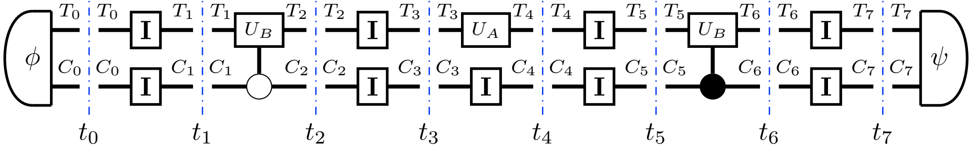
where ϕ and ψ refer to the initial state preparation and final measurement respectively. This also clarifies the scope of the existing no-go results [49–51][52]: they apply only once a particular fragmentation has been fixed, because they exclude precisely *slot-preserving* transformations. Concretely, the no-go statements rule out transformations that keep the slots identities—i.e. the boundary partitions into "past", "future", and intermediate interfaces—fixed while attempting to interchange which agent's operation is represented as local versus time-delocalized. By contrast, adopting a new time frame (or, more generally, a new foliation) amounts to replacing this boundary identification by another, which generically requires a *global* reshuffling of which degrees of freedom are assigned to "past", "future", and intermediate interfaces. Accordingly, the unitaries in Eq. (11)—which act only *within fixed slots*—necessarily preserve the original labels and boundary partition, and therefore cannot implement a genuine change of time frame: they may change the representation *conditional on* a fixed foliation, but they do not transform the foliation (the time-reference data) itself.

Recently, in Ref. [46] the authors considered other two equivalent circuit representations of the quantum switch—representing the causal perspective of Alice and Bob respectively—, with a finer resolution in time



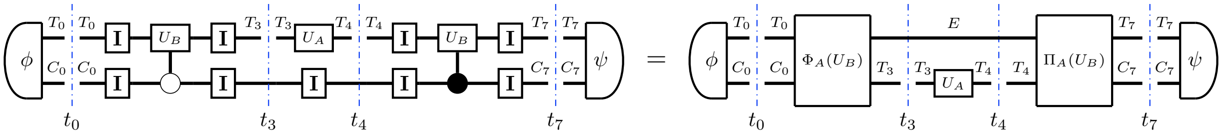
(12)

where additional time steps—identified by $\overset{T_i}{—}\boxed{\mathbf{I}}\overset{T_{i+1}}{—}$ and $\overset{C_i}{—}\boxed{\mathbf{I}}\overset{C_{i+1}}{—}$—are placed between physical events. Analogously to the case discussed in III.1, it is argued that transforming the fragments of one circuit decomposition, like those shown below



(13)

onto those of the other through a class of unitary maps is actually impossible. Following our line of reasoning, the time data labeling the interfaces between the circuit fragments is preserved by the unitary mapping in Eq. (11), thereby preventing a proper transformation into the new time foliation. As a matter of fact we can reproduce the same fragments in the previous section by contracting over T_1, T_2, T_5, T_6 and $C_1, \ldots, C_6$, i.e erasing the corresponding boundaries and time data in Fig. (13), obtaining then

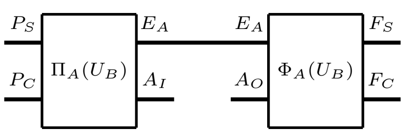


. (14)

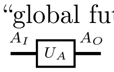
The resulting fragment corresponds to Bob's delocalized intervention as described in Alice's causal frame. We will see in next Sec. III.2 that by using the frame-change transformation we can transform these fragments to those according to Bob's time foliation, like in Eq. (18), where Bob's gate becomes local, while Alice's operation, together with both the state preparation and the measurement, are transformed into a new circuit fragment.

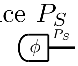
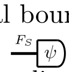
III.2. Unitary time frame-change transformation for quantum switch

With the distinction between (i) a mere coordinate parametrization of a perspective-neutral process and (ii) a foliation into operational fragments in place, we now construct an explicit time-frame change for the quantum switch. We work throughout with the foliation associated with Alice's description in Fig. 10. In that foliation, Bob's operation is time delocalized as illustrated in the following figure

$|W(U_B)\rangle\rangle =$ 

$= |00\rangle\rangle^{F_C P_C} |I\rangle\rangle^{A_I P_S} |U_B\rangle\rangle^{F_S A_O} + |11\rangle\rangle^{F_C P_C} |I\rangle\rangle^{F_S A_O} |U_B\rangle\rangle^{A_I P_S}$. (15)

In Alice's foliation, Bob's gate appears in a coherent superposition of two temporal placements: one in the "global past" segment (t_0, t_1) with input–output systems (P_S, A_I), and one in the "global future" segment (t_2, t_3) with input–output systems (A_O, F_S). By contrast, Alice's own operation $|U_A\rangle\rangle^{A_O A_I} =$  is local in her foliation, confined to the interval (t_1, t_2).

Our goal is to pass to a foliation in which Bob's gate is local. As emphasized in Sec. III.1, this cannot be achieved by any *slot-preserving* transformation, i.e. any unitary that keeps Alice's interface identities (and hence her past/future partition) fixed while attempting to reassign locality/delocalization. Indeed, Eq. (15) shows that "localizing" Bob necessarily requires a reassignment of the delocalized boundary subsystems P_S and F_S, and therefore a concomitant re-identification of which degrees of freedom constitute the relevant boundary slices. In particular, since P_S and F_S sit at the original boundaries, any such reassignment must also transform the associated preparation  and final measurement .

The key ingredient is therefore to implement a *global* reshuffling of the foliation data. Operationally, this is achieved by employing a coherently controlled unitary applied at a *single* interface: because the control is localized at one slice, the induced transformation can act nontrivially on the target degrees of freedom belonging to other slices and thereby reassign the global past/future partition. This stands in contrast with the constructions ruled out by Eq. (11) where transformations are distributed over distinct interfaces and thus preserve the original slot identities.

A unitary that realizes such a time-frame change is

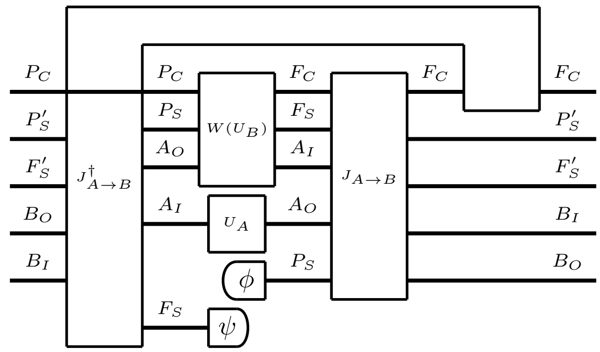
$$|J_{A\to B}\rangle\rangle = |00\rangle\rangle^{F_C F_C} |I\rangle\rangle^{B_O F_S} |I\rangle\rangle^{B_I A_O} |I\rangle\rangle^{F'_S A_I} |I\rangle\rangle^{P'_S P_S} + |11\rangle\rangle^{F_C F_C} |I\rangle\rangle^{B_O A_O} |I\rangle\rangle^{B_I P_S} |I\rangle\rangle^{P'_S A_O} |I\rangle\rangle^{F'_S F_S}. \tag{16}$$

Defining the collections of systems

$$X := F_C F_S A_I A_O P_S, \qquad Y := F_C P_S A_O A_I F_S,$$

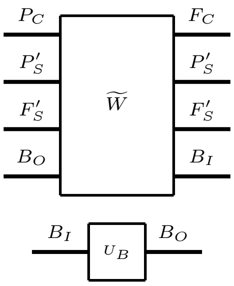
the transformed fragments are obtained by

$$\langle\langle I|^{XX}\langle\langle I|^{YY}\,|J_{A\to B}\rangle\rangle\,|W(U_B)\rangle\rangle\,|U_A\rangle\rangle\,|\phi\rangle\rangle\,|\psi\rangle\rangle\,|J^{\dagger}_{A\to B}\rangle\rangle = \tag{17}$$



$$= |U_B\rangle\rangle^{B_OB_I}\Big(|00\rangle\rangle^{F_CP_C}|I\rangle\rangle^{F'_SP'_S}|U_A\rangle\rangle^{B_IF'_S}|\phi\rangle^{P'_S}|\psi\rangle^{B_O} + |11\rangle\rangle^{F_CP_C}|I\rangle\rangle^{F'_SP'_S}|U_A\rangle\rangle^{P'_SB_O}|\phi\rangle^{B_I}|\psi\rangle^{F'_S}\Big)$$

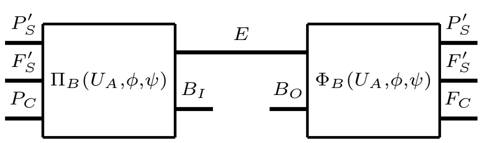
$=$



$$= |U_B\rangle\rangle^{B_OB_I}|\widetilde{W}\rangle\rangle^{F_CF'_SP'_SB_IP_CP'_SF'_SB_O}, \tag{18}$$

where the line crossing the transformation slot denotes a system that is invariant under the perspective change. The resulting process $\widetilde{W}$ admits a circuit representation in which the time-delocalization of the relevant target subsystems is explicit:

$|\widetilde{W}\rangle\rangle =$



$$= \langle\langle I|^{EE}\,|\Phi_A(U_A,\phi,\psi)\rangle\rangle\,|\Pi_A(U_A,\phi,\psi)\rangle\rangle, \tag{19}$$

where, writing $E \coloneqq E_SE_C$,

$$|\Phi_A(U_A,\phi,\psi)\rangle\rangle = |00\rangle\rangle^{E_CP_C}|I\rangle\rangle^{E_SP'_S}|U_A\rangle\rangle^{B_IF'_S} + |11\rangle\rangle^{E_CP_C}|I\rangle\rangle^{E_SP'_S}|\psi\rangle^{B_I}|\phi\rangle^{P'_S}, \tag{20}$$

$$|\Pi_A(U_A,\phi,\psi)\rangle\rangle = |00\rangle\rangle^{F_CE_C}|I\rangle\rangle^{F'_SE_S}|\phi\rangle^{P'_S}|\psi\rangle^{B_O} + |11\rangle\rangle^{F_CE_C}|I\rangle\rangle^{F'_SE_S}|U_A\rangle\rangle^{P'_SB_O}. \tag{21}$$

Two structural features are worth stressing. First, unlike in the slot-preserving scenario, both $J_{A\to B}$ and $J^{\dagger}_{A\to B}$ are controlled by F_C at a *single* interface (Alice's future slice in Fig. 10), while their action globally reassigns the temporal placement of the target subsystems; this is precisely what enables a nontrivial change of foliation. Second, because the transformation necessarily re-identifies the past/future partition, the transformed fragments of the preparation, the final measurement, and Alice's gate (collected into $\widetilde{W}$) do *not* coincide with those appearing in Bob's CRF under the *same* boundary identification: in the new foliation, the degrees of freedom associated with the original preparation/measurement straddle the new past–future partition and thus become time-delocalized. Equivalently, in the absence of further reference structure, a unitary time-frame change that localizes Bob's operation unavoidably alters the global notion of "past" and "future".

This provides the methodological point that aligns with the preceding section. CRF/TDS representations remain equivalent as coordinate parametrizations of the underlying perspective-neutral process. However, once one fixes a foliation into fragments and asks for a *time-frame change*, the relevant symmetry notion must allow transformations that act not only on system degrees of freedom *within* slots but also on the foliation data itself (i.e. on the interface identities and the induced boundary partition). The construction above achieves exactly this: it implements a unitary change of foliation that renders Bob local, at the price of reshuffling the global past/future identification. In the next step of our analysis, we will supply additional reference systems that provide a background circuit fragment (a spatiotemporal scaffold), allowing complementary perspectives to be unitarily related *while preserving* a shared global past and future.

IV. SPACETIME FRAME FOR PROCESS MATRICES

So far we have analyzed the quantum switch at the level of a chosen *time foliation*: we selected interfaces—"cuts"—that single out the laboratories' interventions and the preparation/measurement boundaries, and we treated the resulting interface structure as *frame data* specifying an agent-dependent ordering of events. Crucially, however, the bare switch process contains only the degrees of freedom associated with the laboratories' input–output systems. It therefore provides no additional subsystem that could serve as an *external spatiotemporal background*—no clock/rod register relative to which all events could be localized. In this sense, the previous time-foliated description fixes an ordering, but it still lacks an embedding of the causal network into a common spatiotemporal scaffold.

The aim of this section is to supply precisely this missing structure by extending the process with explicit quantum reference systems (clocks and, where relevant, rods) that carry operationally accessible spatiotemporal information. These additional reference systems provide a *background circuit fragment* shared across perspectives, i.e. a spatiotemporal scaffold into which the process embeds. We will show that, in this extended description, one can implement unitary changes of spatiotemporal perspective that relate complementary CRF/TDS viewpoints *while preserving a common global past and future*. This yields a concrete handle on how the quantum switch–type correlations can be understood as arising from bona fide dynamics of physical systems—clocks/rods—embedded in spacetime.

IV.1. Quantum switch-like process in a general relativistic setting

We now consider a general relativistic representation of a quantum switch–like process. This digression further clarifies the previous results and indicates how to extend the process structure, so that a relational spatiotemporal description can be employed. Specifically, we examine a superposition of two inequivalent physical configurations in which the causal order between two events, A and B, is indefinite: in one branch of the superposition, B lies in the past of A, namely ($B \prec A$), while in the other branch, the order is reversed ($A \prec B$). This scenario is illustrated in Fig. 1.

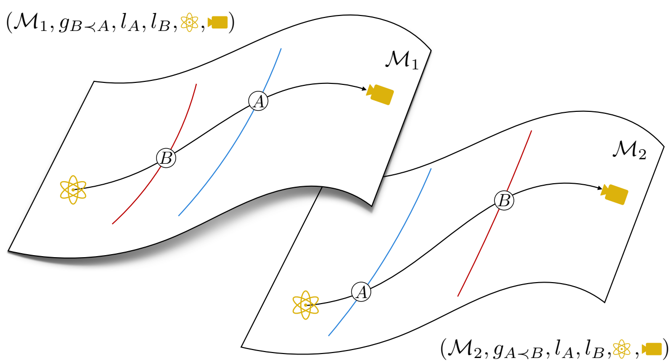


Figure 1. **Superposition of physically inequivalent configurations**—The events A and B are given by the intersection points of the world-lines of the target system (black arrow), Alice's laboratory (blue line), and Bob's laboratory (red line). The state preparation and the final measurement are represented by the yellow atom-shaped symbol and the detector symbol, respectively.

Each configuration is specified by the tuple of the type $(\mathcal{M}_1, g_{B\prec A}, l_A, l_B, ⚛, ■)$ which encodes: the space of parameters—the manifold $\mathcal{M}$ and the physical degrees of freedom, here consisting of the gravitational field, the world-lines of Alice's and Bob's laboratories, the target system, and the detector performing the final measurement.

The underlying physics is invariant under any change of coordinate system, implemented by the group of diffeomorphisms on the manifold. In the regime considered, the standard notion of symmetry is extended to include quantum-controlled symmetry transformations and we refer to the resulting group as the *quantum symmetry group* [12, 16, 17]. This extension allows us to adopt a physical perspective by parametrizing spacetime by means of a suitable set of scalar fields whose configuration defines what we call *quantum coordinates*, or equivalently *quantum reference frames* (QRFs) [53]. In the chosen "quantum coordinate chart", Alice's laboratory is local, while Bob's and the target's worldlines are spacetime-delocalized. Nevertheless, event A remains local because the superposed target worldlines intersect Alice's worldline at the same point of the quantum coordinate chart. Moreover, both branches of the superposition share the same notion of global past and future, making the initial state preparation and final measurement local as well.

This relational description in terms of quantum coordinates can be be achieved via a suitable quantum-controlled diffeomorphism $\phi := (\phi_1, \phi_2)$, where $\phi_1 : \mathcal{M}_1 \to \mathcal{M}_A$ and $\phi_2 : \mathcal{M}_2 \to \mathcal{M}_A$ act on $\mathcal{M}_1$ and $\mathcal{M}_2$ respectively. The physical configuration is now specified by the tuple $(\mathcal{M}_1, g_{B\prec A}, l_A, l_B,$ ⧫, ⚛, ▬$)$, and similarly for $\mathcal{M}_2$, which explicitly includes the reference degrees of freedom–i.e. ⧫. In Fig. 2, these quantum coordinates are depicted as the gray grid, providing the spatiotemporal coordinatization relative to which events and worldlines are defined.

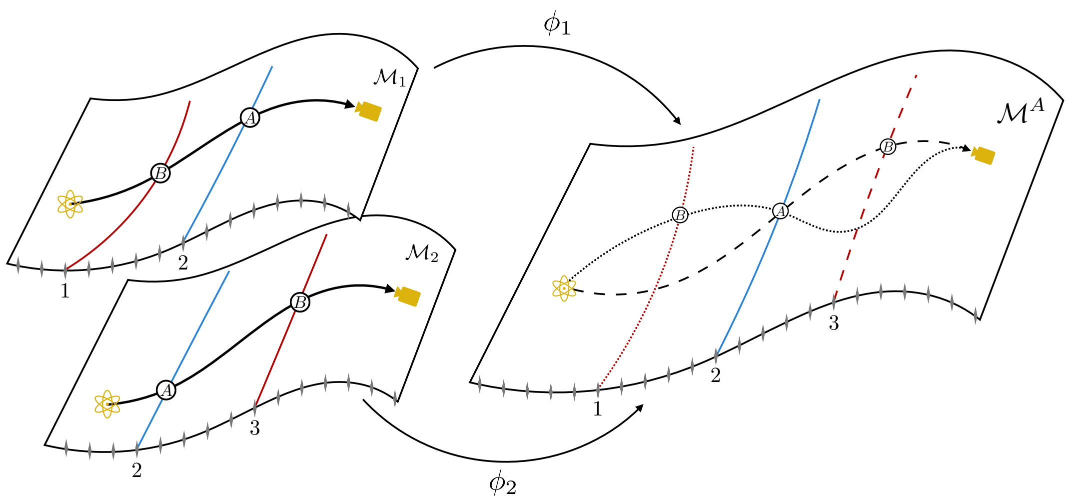


Figure 2. **Quantum coordinate transformation**—The quantum-controlled diffeomorphism allows to take the perspective of the physical scaffolding—i.e. quantum coordinates—depicted as the gray grid. The transformed state on the right is expressed in those quantum coordinates, such that Alice's worldline is localized—configuration 2—, whereas Bob's worldline is spacetime delocalized—i.e., in superposition of configurations 2 and 3.

The ability to adopt the complementary physical perspective—in which event B is local while A is delocalized—depends solely on the availability of another physical frame prepared in an appropriate configuration. When such a frame is available—i.e. ⧫—both perspectives can be realized, as illustrated in Fig. 3.

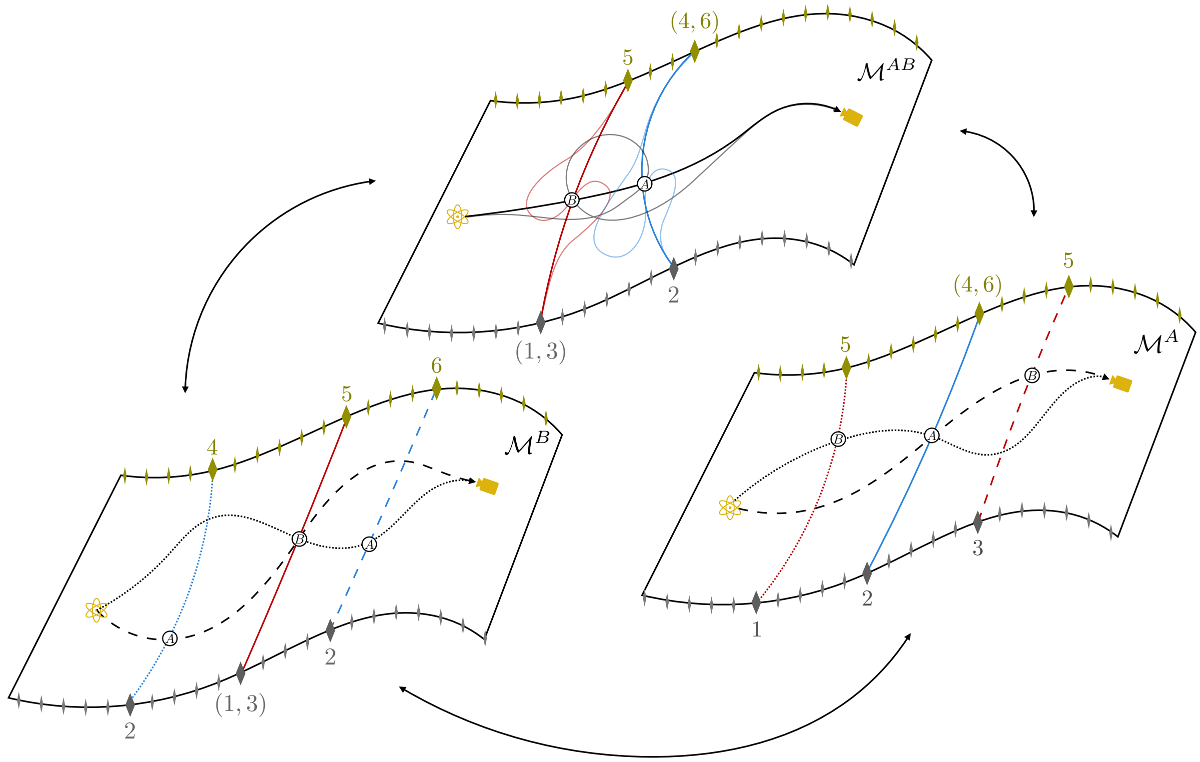


Figure 3. **Equivalent quantum coordinate representations**—Three different coordinate representations of the same process are shown. (Top) process matrix representation, where all events are local; (left) Bob's causal reference frame; (right) Alice's causal reference frame. One can change representation via suitable quantum symmetry transformation.

The equivalence of these coordinate representations Fig. 3 mirrors the equivalence between CRF and TDS descriptions. The top configuration in Fig. 3 corresponds to the abstract representation—process matrix—of the quantum switch, where each gate, i.e. event, appears in a single slot of the process. The remaining two configurations correspond to Alice's and Bob's CRFs decompositions respectively, as in Eqs. (3) and (8). The equivalence underlined by Fig. 3 do not corresponds though to the equivalence of Eq. (8), that is an operational equivalence of the two processes that provide the same statistic, rather than a map connecting different perspectives on the same process.

The possibility to switch from the different perspectives displayed in Fig. 3 is granted by the presence of suitable QRFs. In particular, obtaining a QRF perspective that matches one of the considered configurations depends entirely on the physical state of the QRF—specifically, on whether it is suitably correlated with the rest of the systems [17].

In Sec. III.2, however, beyond the clock's readings in Eqs. (10) and (21), the scenario includes only the target system and the control, which may be identified with the gravitational field in the general-relativistic setting, and no QRFs. Consequently, what is described is merely the time delocalization of a single gate and not the spacetime delocalization of the corresponding worldline.

IV.2. Spacetime formulation of a switch-like process

The physical situation that we are aiming to reproduce is intuitively illustrated in Fig. 4, where the events and the corresponding worldliness are parametrized with respect to a physical scaffolding—e.g. suitable set of scalar fields—, providing a notion of rod and clock.

Motivated by this spatiotemporal representation, we introduce additional structure into the circuit model of the quantum switch—an extra system beyond the target and the control—implemented so as to supply the missing spatial dimensionality. As a matter of fact a third system allows for a diagrammatical representation of the both the geodesics involved in the full spacetime picture. This is realized by the circuit fragment shown below, which specifies the

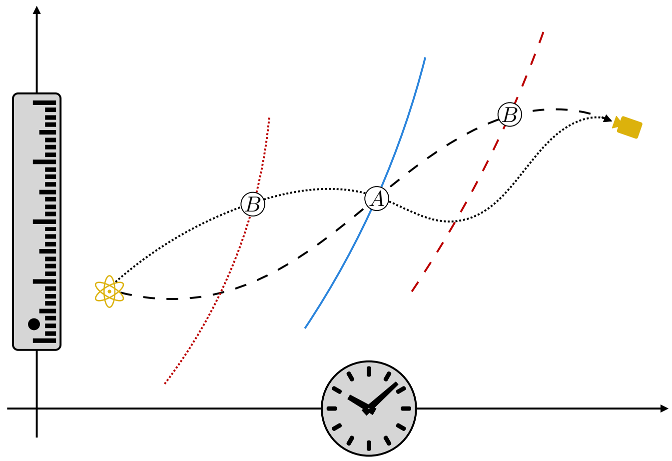
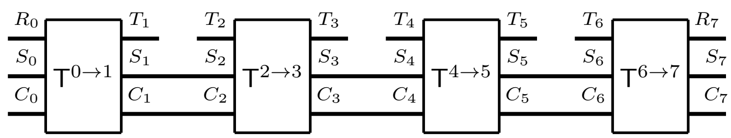


Figure 4. **Spatiotemporal QRF**—Physical systems serve as rods and clocks, providing a spatiotemporal resolution of the process. The resulting relational picture is free of external parameters.

"spatiotemporal background" with respect to which the process unfolds

$$|W_{B|A}\rangle\!\rangle = \quad (22)$$



where the single boxes defining the time steps of the process are

$$|\mathsf{T}^{0\to1}\rangle\!\rangle = |00\rangle\!\rangle^{C_1C_0}|I\rangle\!\rangle^{S_1R_0}|I\rangle\!\rangle^{T_1S_0} + |11\rangle\!\rangle^{C_1C_0}|I\rangle\!\rangle^{T_1R_0}|I\rangle\!\rangle^{S_1S_0}, \quad (23)$$

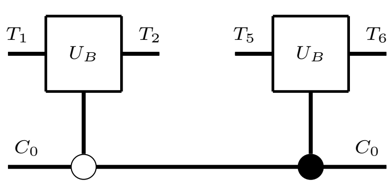
$$|\mathsf{T}^{1\to2}\rangle\!\rangle = |00\rangle\!\rangle^{C_3C_2}|I\rangle\!\rangle^{T_3T_2}|I\rangle\!\rangle^{S_3S_2} + |11\rangle\!\rangle^{C_3C_2}|I\rangle\!\rangle^{T_3S_2}|I\rangle\!\rangle^{S_3T_2}, \quad (24)$$

$$|\mathsf{T}^{4\to5}\rangle\!\rangle = |00\rangle\!\rangle^{C_5C_4}|I\rangle\!\rangle^{S_5T_4}|I\rangle\!\rangle^{T_5S_4} + |11\rangle\!\rangle^{C_5C_4}|I\rangle\!\rangle^{T_5T_4}|I\rangle\!\rangle^{S_5S_4}, \quad (25)$$

$$|\mathsf{T}^{6\to7}\rangle\!\rangle = |00\rangle\!\rangle^{C_7C_6}|I\rangle\!\rangle^{S_7S_6}|I\rangle\!\rangle^{R_7T_6} + |11\rangle\!\rangle^{C_7C_6}|I\rangle\!\rangle^{S_7T_6}|I\rangle\!\rangle^{T_7S_6}, \quad (26)$$

and the three open slots corresponds to the spacetime locations of Bob's and Alice's events. Let us note that S and C label the target and control systems, respectively, while R denotes the additional reference system. Now we provide the agent's operations, starting from the QRF perspective according to which Bob's operation is spacetime delocalized, namely

$$|\mathbf{U}_{B|A}(U_B)\rangle\!\rangle =$$



$$= |00\rangle\!\rangle^{C_0C_0}|U_B\rangle\!\rangle^{T_2T_1}|I\rangle\!\rangle^{T_6T_5} + |11\rangle\!\rangle^{C_0C_0}|I\rangle\!\rangle^{T_2T_1}|U_B\rangle\!\rangle^{T_6T_5}, \quad (27)$$

While Alice's operation occurs at time step $3\to4$, namely $|U_A\rangle\!\rangle^{T_4T_3} = \overset{T_3}{—}\boxed{U_A}\overset{T_4}{—}$. Let us notice that upon placing the operations in the corresponding slots of the circuit we have

$$\langle\!\langle \mathbf{U}_{B|A}(U_B)|\langle\!\langle U_A||W_{B|A}\rangle\!\rangle = |I\rangle\!\rangle^{R_7R_0}|W_{SW}(U_A,U_B)\rangle\!\rangle \quad (28)$$

where W corresponds to the quantum switch. Hence, discarding the extra system R_7, while plugging any deterministic state in R_0, does not cause any loss of coherence. This guarantees statistical equivalence between the present process and the standard quantum switch.

V. TRANSFORMATIONS BETWEEN SPATIOTEMPORAL PERSPECTIVES

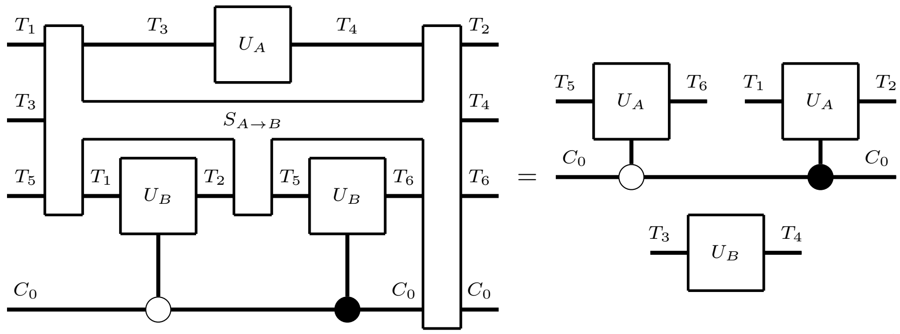
Now we discuss how to change perspective. We seek a QRF perspective in which the background fragment places Bob's operation at a well-defined spacetime location, i.e. a single circuit slot. In this frame, Bob's intervention is represented by a single non-controlled gate, whereas Alice's becomes delocalized—entangled with the control system. Below we show a unitary map that transforms the fragments accordingly:

$$|S_{A\to B}\rangle\!\rangle = |00\rangle\!\rangle^{C_0C_0}|I\rangle\!\rangle^{T_4T_2}|I\rangle\!\rangle^{T_1T_3}|I\rangle\!\rangle^{T_2T_4}|I\rangle\!\rangle^{T_3T_1}|I\rangle\!\rangle^{T_5T_5}|I\rangle\!\rangle^{T_6T_6} \tag{29}$$

$$+ |11\rangle\!\rangle^{C_0C_0}|I\rangle\!\rangle^{T_4T_6}|I\rangle\!\rangle^{T_5T_3}|I\rangle\!\rangle^{T_6T_4}|I\rangle\!\rangle^{T_3T_5}|I\rangle\!\rangle^{T_2T_2}|I\rangle\!\rangle^{T_1T_1}. \tag{30}$$

Let us now define $\mathsf{K} := C_0T_1T_2T_3T_4T_5T_6$ and transform Alice and Bob's fragments of the circuit

$$\langle\!\langle I|^{\mathsf{KK}}|S_{A\to B}\rangle\!\rangle|\mathbf{U}_{B|A}(U_B)\rangle\!\rangle|U_A\rangle\!\rangle =$$



(31)

$$= |\mathbf{U}_{A|B}(U_A)\rangle\!\rangle|U_B\rangle\!\rangle, \tag{32}$$

which in the current notation is

$$|\mathbf{U}_{A|B}(U_A)\rangle\!\rangle^{C_0T_2T_4T_6C_0T_1T_3T_5}|U_B\rangle\!\rangle^{T_4T_3} = \left(|00\rangle\!\rangle^{C_0C_0}|U_A\rangle\!\rangle^{T_2T_1}|I\rangle\!\rangle^{T_6T_5} + |11\rangle\!\rangle^{C_0C_0}|I\rangle\!\rangle^{T_2T_1}|U_A\rangle\!\rangle^{T_6T_5}\right)|U_B\rangle\!\rangle^{T_4T_3}. \tag{33}$$

As expected, the transformed fragments satisfy the desiderata: Bob's operation is disentangled from the control and localized at the time step $T_3 \to T_4$, whereas Alice's operation is delocalized between $T_5 \to T_6$ and $T_2 \to T_3$, being entangled with C_0.

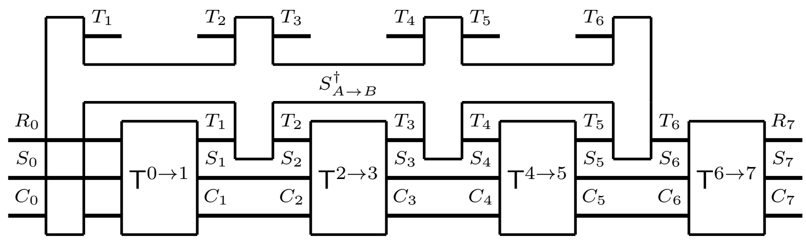
Let us now transformed the fragment of the background circuit

$$\langle\!\langle I|^{\mathsf{KK}}|S^\dagger_{A\to B}\rangle\!\rangle|W_{B|A}\rangle\!\rangle = |00\rangle\!\rangle^{C_7C_0}|I\rangle\!\rangle^{T_3S_0}|I\rangle\!\rangle^{T_5R_0}|I\rangle\!\rangle^{T_1T_4}|I\rangle\!\rangle^{S_7T_2}|I\rangle\!\rangle^{R_7T_6}$$

$$+ |11\rangle\!\rangle^{C_7C_0}|I\rangle\!\rangle^{T_1R_0}|I\rangle\!\rangle^{T_5S_0}|I\rangle\!\rangle^{R_7T_2}|I\rangle\!\rangle^{T_3T_6}|I\rangle\!\rangle^{S_7T_4} = |W_{A|B}\rangle\!\rangle. \tag{34}$$

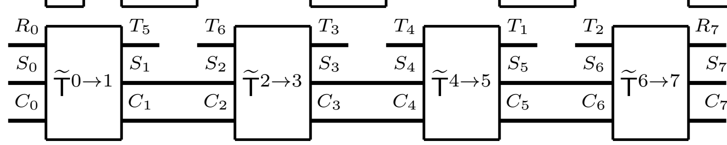
As expected, the fragment of the circuit which correspond to the background structure transforms in such a way that the first and last time slots correspond now to Alice lab while Bob's one is localized between them, namely

$$|S^\dagger_{A\to B}(W_{B|A})\rangle\!\rangle =$$



(35)

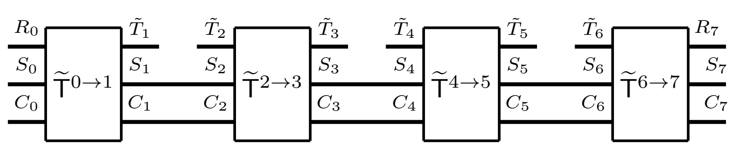
$$=$$



(36)

Then by simply relabeling the system of the new time, such that the first $(\tilde{T}_1, \tilde{T}_2)$ and the third time step $(\tilde{T}_5, \tilde{T}_6)$ correspond to (T_5, T_6) and (T_1, T_2) respectively, we arrive at representation as displayed below

$$|W_{A|B}\rangle\!\rangle =$$



(37)

What we showed is the possibility to transform unitarily the fragments associated with the causal decomposition of Alice (27) and the corresponding background (22) into the those associated to Bob's complementary frame (31) and background (37).

VI. CONCLUSIONS

The starting point of this work is the distinction between coordinates understood as abstract labels in the perspective neutral—unfrgmanted—picture and coordinates understood as frame data, that *can* be instantiated by physical systems—e.g. clocks and rods. Within this conceptual standpoint, we first argue that the CRF and TDS representations of a pure process are best understood as *coordinate parameterizations* of a perspective-neutral higher-order object. Their equivalence is therefore a statement of *coordinate invariance* at the level of the unfragmented process, rather than a relativization of the process with respect to a physical frame.

Our next contribution is methodological. We formalize an agent-dependent time perspective by foliating a chosen CRF/TDS decomposition into circuit fragments. This notion of *operational cut* upgrades the interface labels from bookkeeping parameters to frame data, namely clock-readings associated with the chosen time frame. In this setting, a genuine frame change must co-transform both the circuit fragments and the classical frame data—i.e. the fragment interfaces—so as to effectively re-foliate the process. This requirement is precisely what underlies the no-go result for slot-preserving unitaries [44, 46]: acting locally at each fragment interface necessarily preserves the original frame data, specifically the global past and future, and therefore cannot implement a change of time frame.

Focusing on the quantum switch, we show that although our perspective-change map unitarily transforms one time-foliated circuit into another in which a different agent is local, it generically *does not* preserve a common notion of global past and future—instead, both are reshuffled and mixed once the map is applied. In this way, the time-fragmented TDS decomposition seems need not be unitarily related to its *complementary* decomposition, which *does* share the same global past and future. This may seem paradoxical, since the quantum switch can be physically realized in both gravitational and optical settings [30, 38], where one can choose descriptions in which either agent's operation is local while the other's is nonlocal, yet both descriptions share the same global past and future. How, then, can two physically equivalent situations—each realizable and sharing a common global past and future—fail to be related by a unitary transformation? The question is naturally resolved once one realizes that, in contrast to physical realizations of the quantum switch that include quantum reference frames (QRFs), the bare switch-like process does not contain sufficient reference degrees of freedom to instantiate distinct *spatiotemporal* perspectives that agree on a global past and future. In other words, the process lacks the physical structure required to support a background scaffold relative to which complementary perspectives can be related by a unitary change of frame.

To overcome this limitation, we extend the quantum switch by explicitly adding QRFs, thus supplying a shared *spatiotemporal scaffold* in which the process is embedded. The extended process reproduces the same statistics as the abstract switch, but it now admits a genuinely relational spacetime description. In this extended setting, we exhibit an explicit unitary QRF-change transformation that maps the fragments of one extended TDS—spacetime TDS—to the *complementary* one, while retaining a shared global past and future. This restores, at the level of spatiotemporal frames, the perspective complementarity in TDS/CRF that is not unitarily accessible using a time frame only.

We contend that developing a physical notion of perspective for processes—and a corresponding perspective-change transformation—is directly relevant to the question of empirical realizability. By "realizable" we mean that the coupled dynamics of spacetime and matter admits configurations that yield statistics that are operationally equivalent to those of the abstract—bare—process. In Refs. [54, 55], the authors use graph-theoretic techniques to derive admissibility criteria for causal structures, formulating explicit graph constraints that any spatiotemporal causal structure connecting local quantum laboratories must satisfy. They also propose a general-relativistic generalization of a Bell-type test that can certify the presence of a dynamical causal structure between the parties involved. Moreover, while processes with quantum-controlled causal orders can be realized in general relativistic settings [13, 30, 56], such realization of so-called *unitary noncausal* processes that violate causal inequalities—for example, the *Baumeler–Wolf* (BW) process, may appear to require spacetime solutions with closed timelike curves (CTCs). At the same time, recent work by Wechs, Branciard, and Oreshkov argue that there exits processes violating causal inequalities that nevertheless admit realizations on time-delocalised subsystems within standard quantum theory, i.e., without invoking exotic spacetime structure [39].

We see our work as providing a concrete route toward assessing physical realizability of process matrices through bona fide spacetime-and-matter dynamics. This naturally motivates the next step: extending the same perspective-based construction beyond switch-like processes. In particular, applying this framework to unitary noncausal processes would test whether noncausality can be re-expressed as quantum spatiotemporal delocalization of laboratories without invoking exotic structures, such as CTCs.

Finally, we return to our initial distinction between abstract coordinates and physical scaffolding. We argue that the latter is crucial for implementing a proper transformation from one perspective to another. Does this imply, however, that it is impossible to establish physical equivalence between two perspectives without invoking physical clocks and rods (i.e., specific particles and fields)? This may seem to contradict the notion of coordinates in GR, where coordinates provide a chart on regions of the manifold and are, conceptually, prior to the actual fields defined on it. Our understanding is that this is not necessary: the physical scaffolding can be understood as *potential* or *gedanken* matter fields, not necessarily the ones that are implemented. With this understanding, the situation is not very different from classical GR: one may choose different potential coordinate fields to express observables, and then show that no observable depends on that choice. However, when mapping from one choice of potential fields to another, one must still include them in the overall transformation in order to preserve unitarity.

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