get ready to learn
the ZX-calculus

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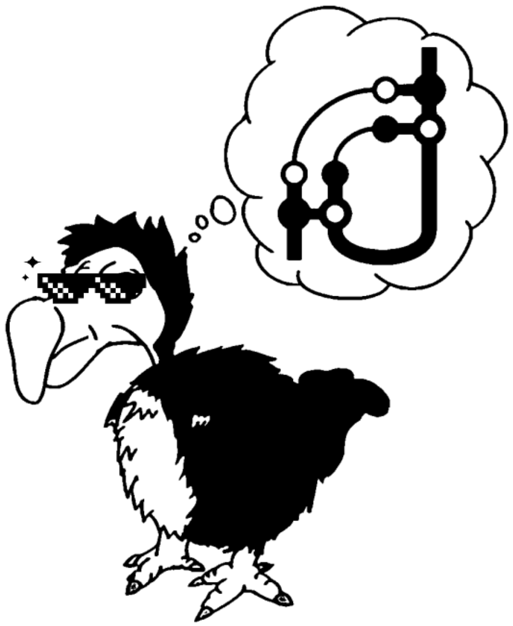
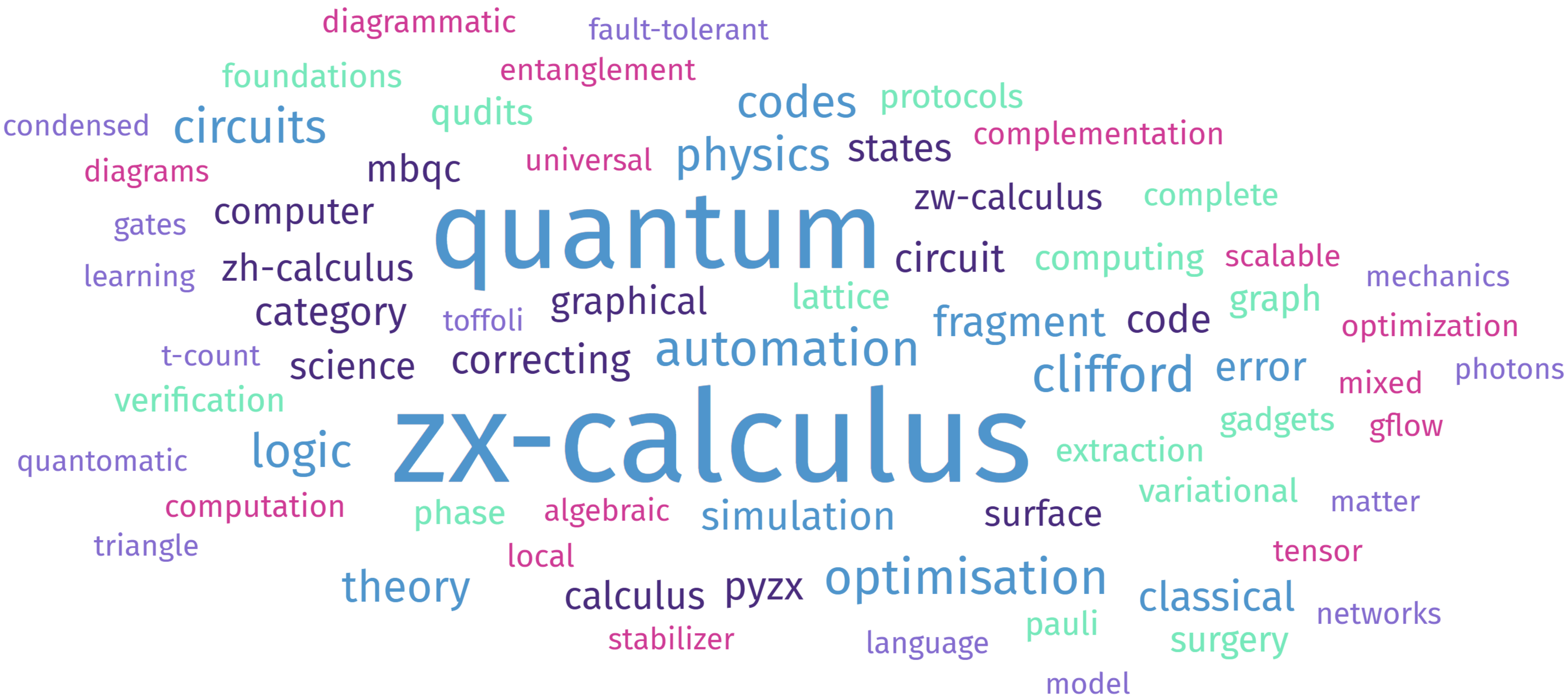


August 15, 2026
Quantum Physics and Logic Conference

Overview

- Part 1 of the tutorial covered:
  - Process theories and ZX-calculus
  - Circuit Optimization and Classical Simulation
  - Measurement-based quantum computation
- Part 2 will cover:
  - Quantum Error Correction
  - Higher dimensional variants of ZX calculus



Word cloud of top 70 words from zxcalculus.com/publications

ZX-calculus research areas

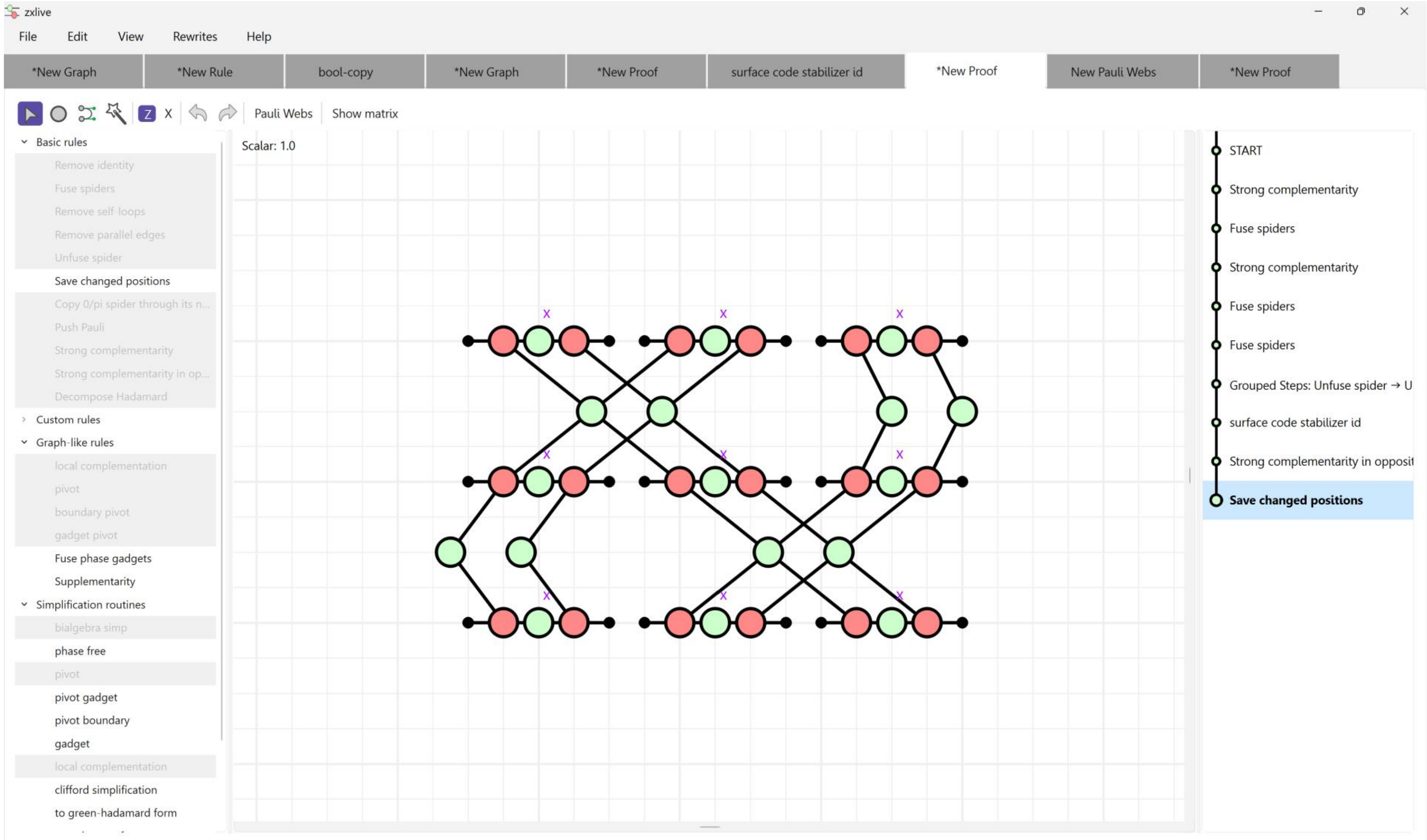
and examples at Quantum Physics and Logic Conference 2026

Quantum software

Ex. PyZX, QuiZX, ZXLive

ZXLive Tutorial *Mon 17:30-18:30*

Boldizsár Poór and Razin Shaikh



Classical simulation

- *Mon 16:15-16:40 Parallel C* Efficient classical simulation of low-rank-width quantum circuits using ZX-calculus Fedor Kuyanov, Aleks Kissinger
- *Mon 16:40-17:05 Parallel C* Simulating magic state cultivation with few Clifford terms Kwok Ho Wan, Zhenghao Zhong, Ainhoa Zapirain

Measurement-based quantum computing and flow

- *Wed 9:30-10:15 Plenary* Completeness for flow-preserving rewrite rules / Generating one-way computations with flow: flow-preserving rewriting that ignores the interpretation Miriam Backens, Simon Perdrix / Miriam Backens
- *Wed 14:55-15:20 Parallel B* Working with measurement-based computations on qudits Piotr Mitosek, Miriam Backens
- *Wed 15:20-15:45 Parallel B* ZX-flow: a flexible criterion for deterministic computation with ZX-diagrams Aleks Kissinger, John van de Wetering

Algebraic geometry

- *Mon 11:55-12:20 Plenary* [Graphical algebraic geometry: from ideals and varieties to qudit ZH completeness] Dichuan Gao, Razin A. Shaikh, Aleks Kissinger
- *Fri 14:30-14:55 Parallel A* [Meromorphic quantum computing] Simon Burton, Hussain Anwar

Quantum error correction

- *Tue 15:20-15:45 Parallel A* The delayed stabiliser ZX-calculus Cole Comfort, Giovanni de Felice
- *Wed 16:40-17:05 Parallel B* A three-way normal form for stabiliser codes across ZX diagrams, circuits, and tableaus Lia Yeh, Jiaxin Huang, Aleks Kissinger, Sarah Meng Li, John van de Wetering
- *Fri 14:30-14:55 Parallel A* Meromorphic quantum computing Simon Burton, Hussain Anwar
- *Fri 14:55-15:20 Parallel A* Transversal AND in quantum codes Christine Li, Lia Yeh

Fault-tolerance

- *Tue 9:30-10:15 Plenary* Completeness for fault equivalence of Clifford ZX diagrams / Fault tolerance by construction Maximilian Rüsch, Aleks Kissinger, Benjamin Rodatz / Benjamin Rodatz, Boldizsár Poór, Aleks Kissinger
- *Tue 14:30-14:55 Parallel A* SpiderCat: optimal fault-tolerant cat state preparation Andrey Boris Khesin, Sarah Meng Li, Boldizsár Poór, Benjamin Rodatz, John van de Wetering, Richie Yeung

Quantum computing for fermions

- *Wed 16:15-16:40 Parallel B* Haytham McDowall-Rose, Razin A. Shaikh, Lia Yeh
  From fermions to qubits: a ZX-calculus perspective
- *Poster* ZW-calculus in q-arithmetic Renaud Vilmart, Vincent Nguyen

Condensed matter physics, quantum machine learning, and more

- *Wed 10:15-11:00 Plenary* [Beyond Penrose tensor diagrams with the ZX calculus: applications to quantum computing, quantum machine learning, condensed matter physics, and quantum gravity] Quanlong Wang, Richard D. P. East, Razin A. Shaikh, Lia Yeh, Boldizsár Poór, Bob Coecke

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Process theories
(symmetric monoidal categories)

A diagram is made of **boxes** and **wires**.

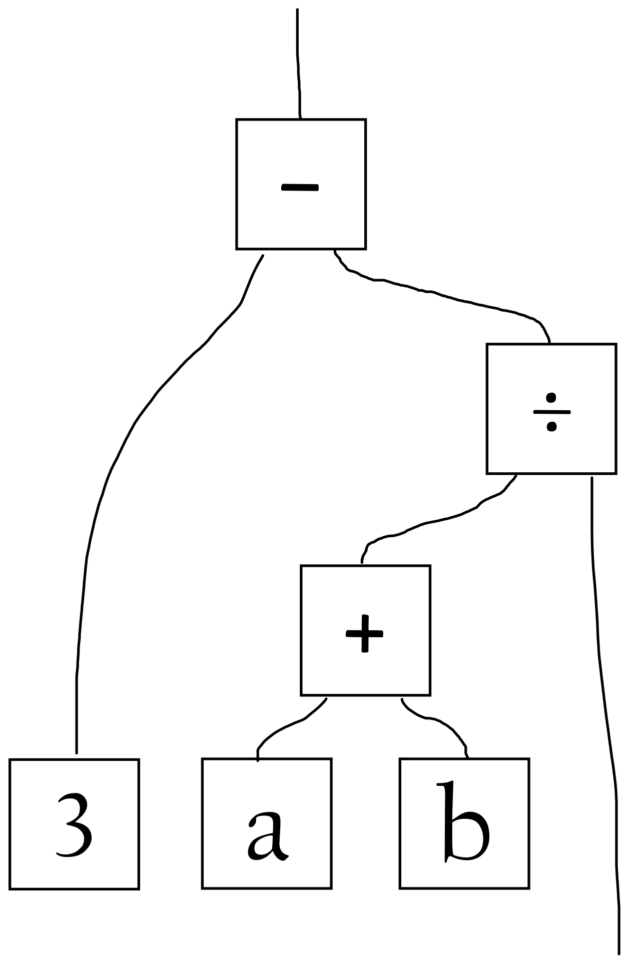
Every **wire** has a type.

Boxes take zero or more wires as input,
and output zero or more wires.

Example: Arithmetic

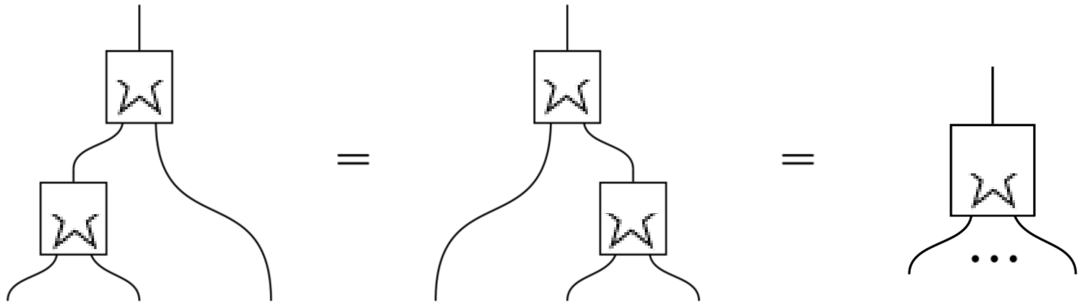
Wires: numbers
Boxes: binary operations

$$y = 3 - ((a + b) \div x) \rightsquigarrow$$

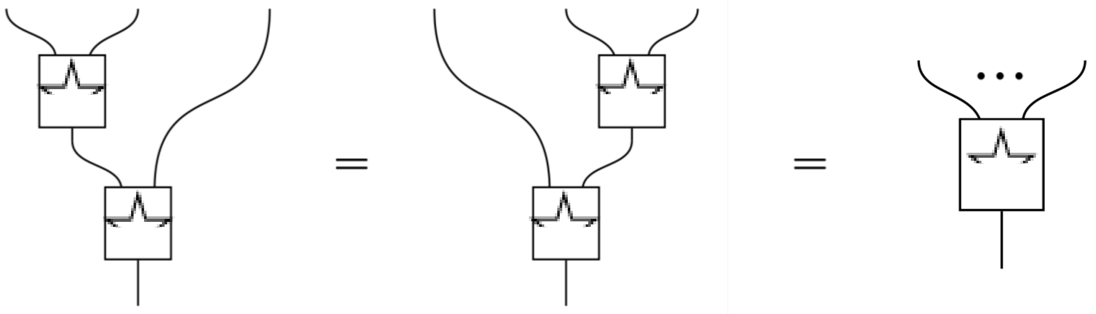


Process theories

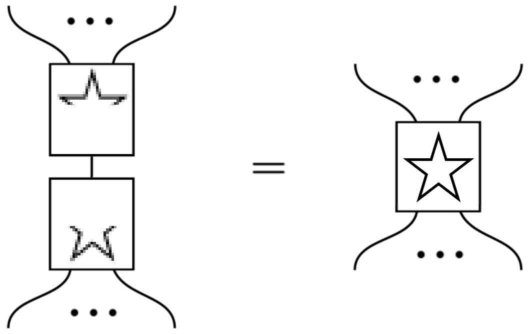
If a box is associative,



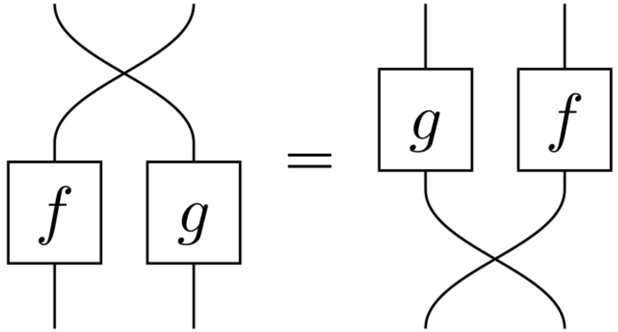
If it is coassociative,

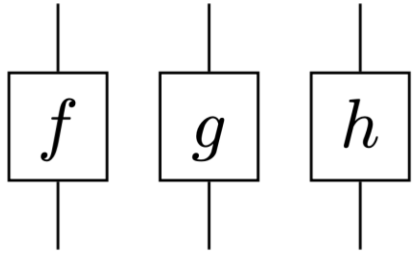


If it is both, then:

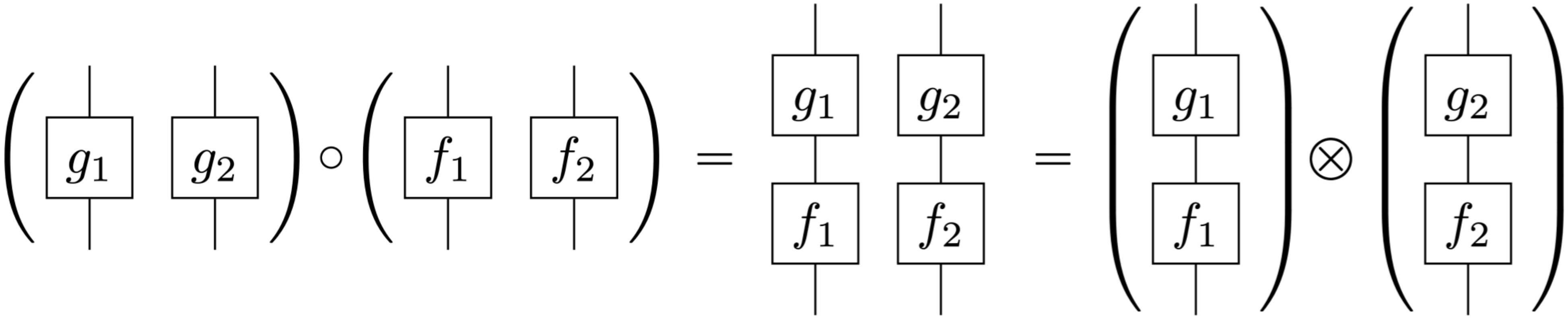


Processes in parallel and in sequence



$(f \otimes g) \otimes h =$ 

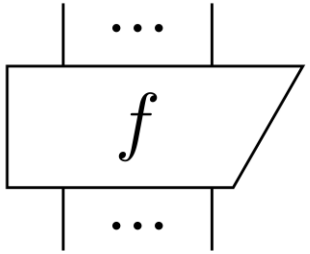
$= f \otimes (g \otimes h)$



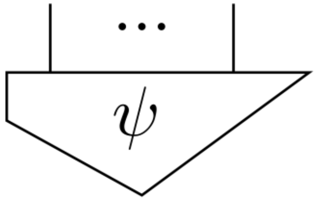
Quantum processes

Wires: quantum information
Boxes: linear maps

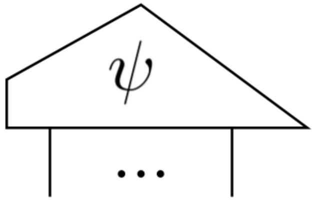
map f $\rightsquigarrow$



state $|\psi\rangle$ $\rightsquigarrow$

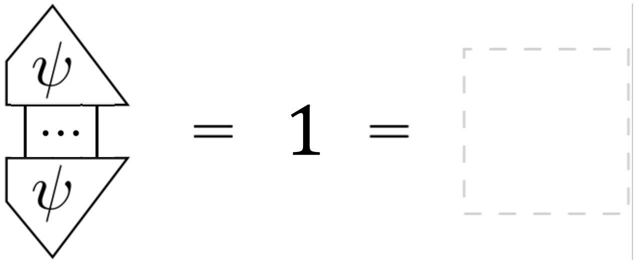


effect $\langle\psi|$ $\rightsquigarrow$



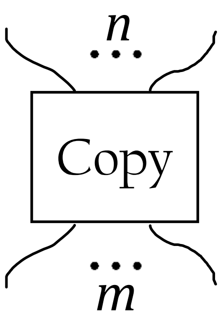
scalar c $\rightsquigarrow$ c

For all normalized states:

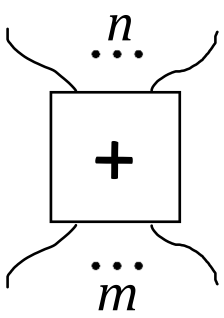


On $|0\rangle$ and $|1\rangle$:

This is Copy



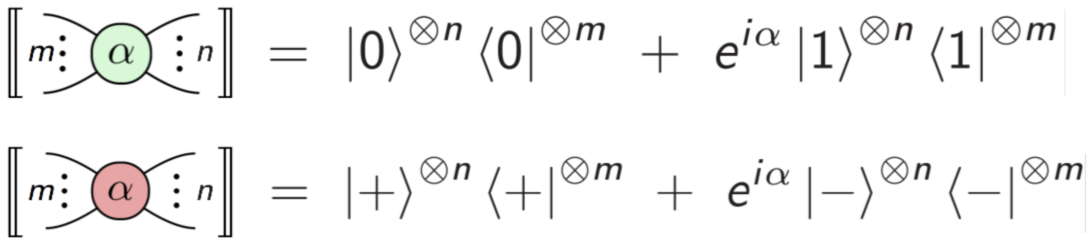
This is Addition



2007: Creation of the ZX-calculus

This is Z

This is X



A <u>diagram</u> in the ZX calculus is an undirected graph where each vertex is Z or X.

Interacting quantum observables. ICALP (2007). Bob Coecke and Ross Duncan.

The ZX calculus, on one qubit

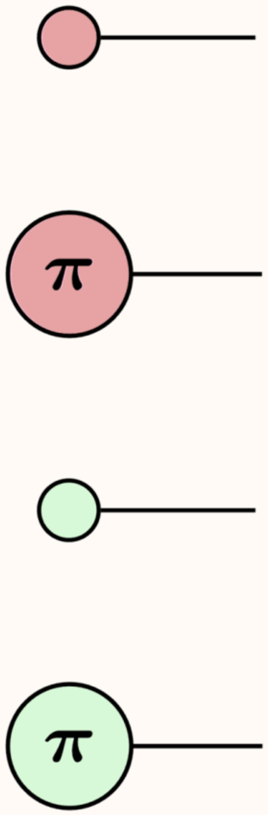
Basis states

$|0\rangle$ — $\rightsquigarrow$

$|1\rangle$ — $\rightsquigarrow$

$|+\rangle$ — $\rightsquigarrow$

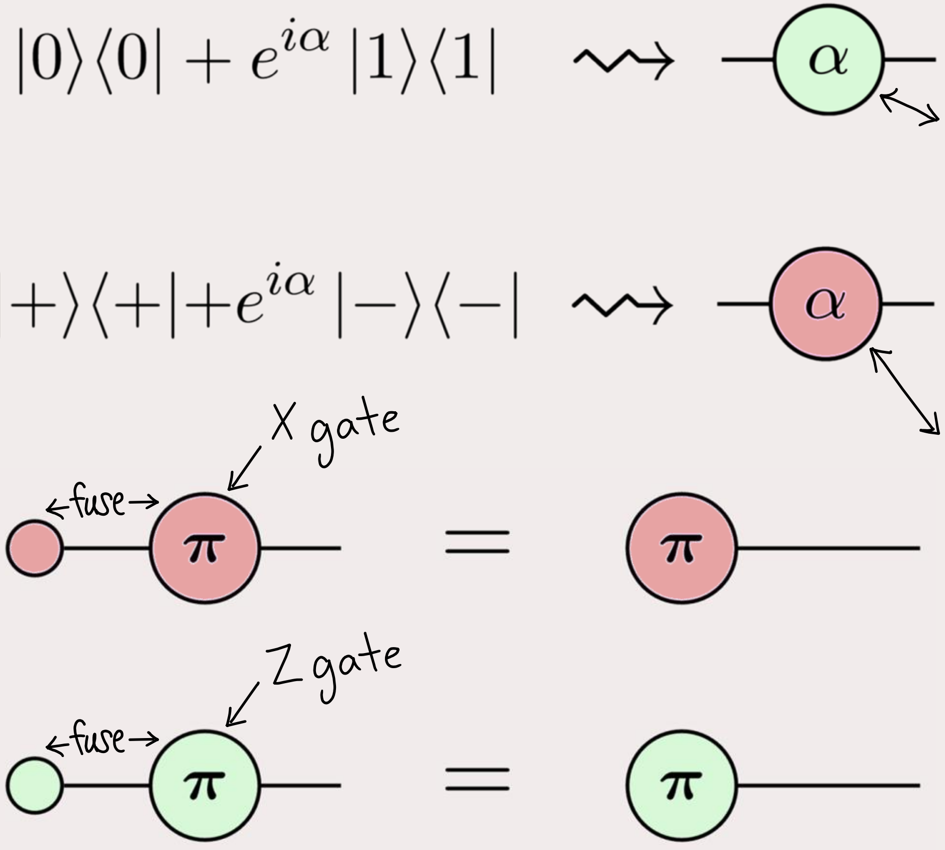
$|-\rangle$ — $\rightsquigarrow$

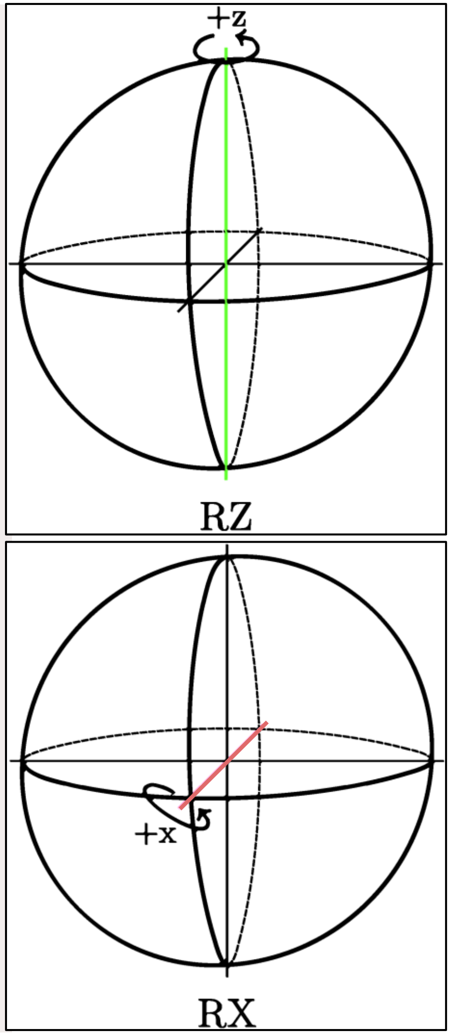


Phase gates

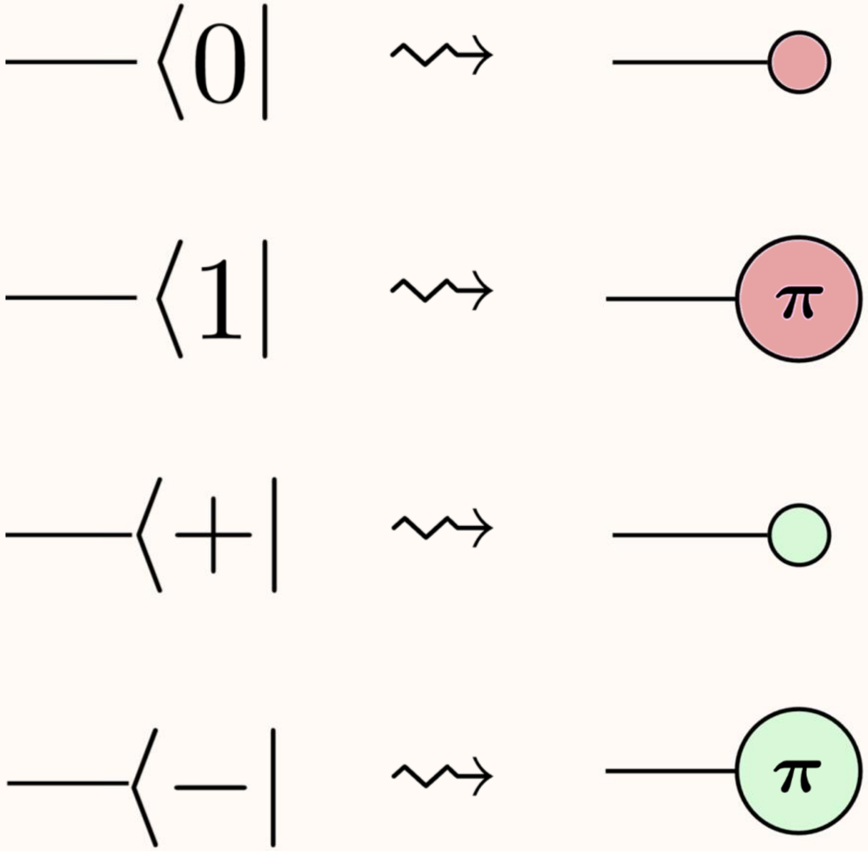
$|0\rangle\langle 0| + e^{i\alpha}|1\rangle\langle 1| \rightsquigarrow$

$|+\rangle\langle +| + e^{i\alpha}|-\rangle\langle -| \rightsquigarrow$





Qubit Measurement Outcomes

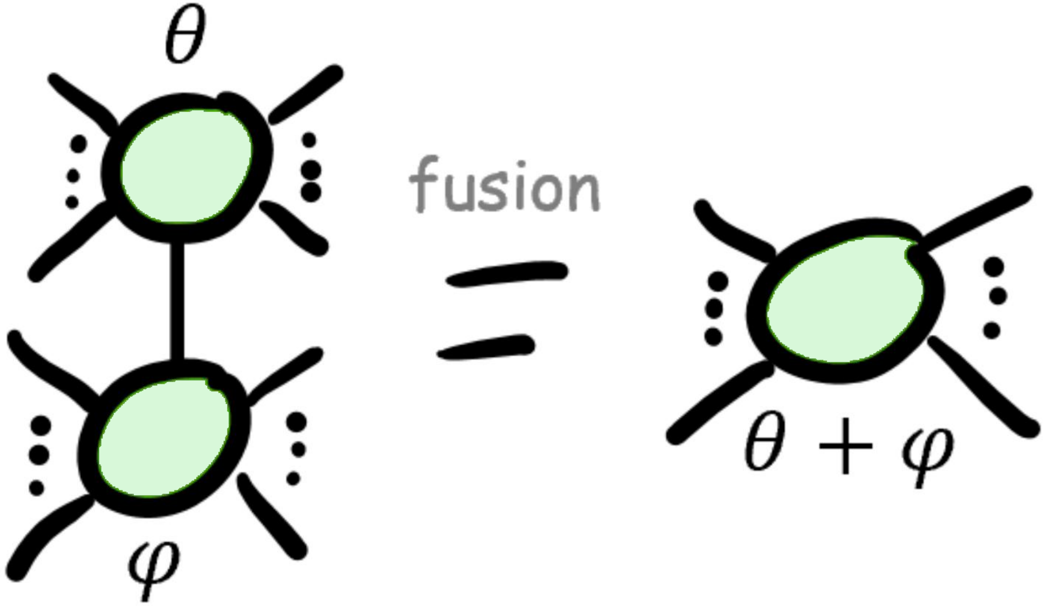


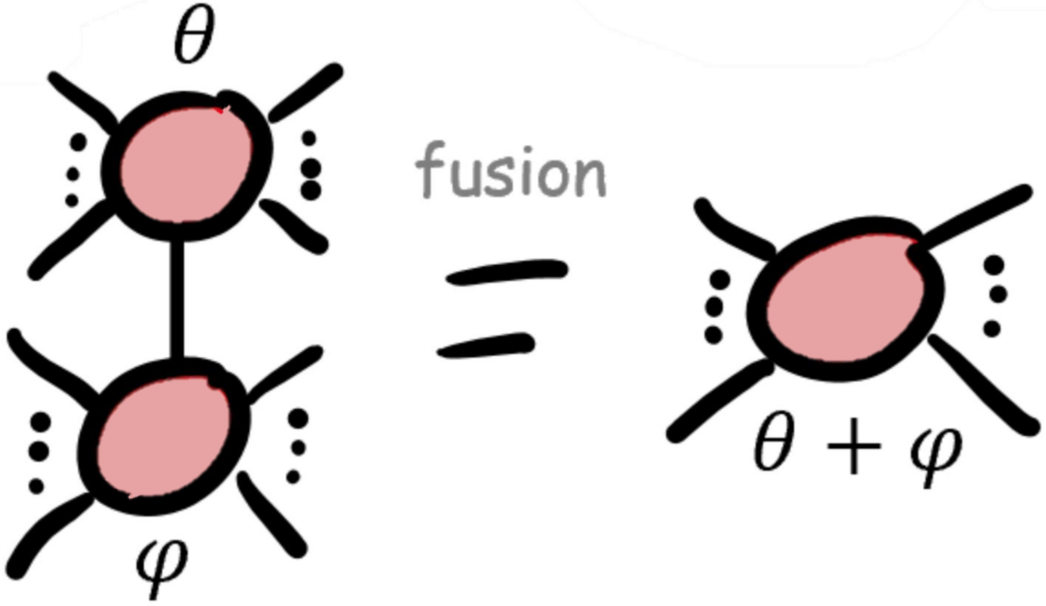
Qubit Measurements

Z basis measurement
with outcome bit $b_1 \in \{0,1\}$

X basis measurement
with outcome bit $b_0 \in \{0,1\}$

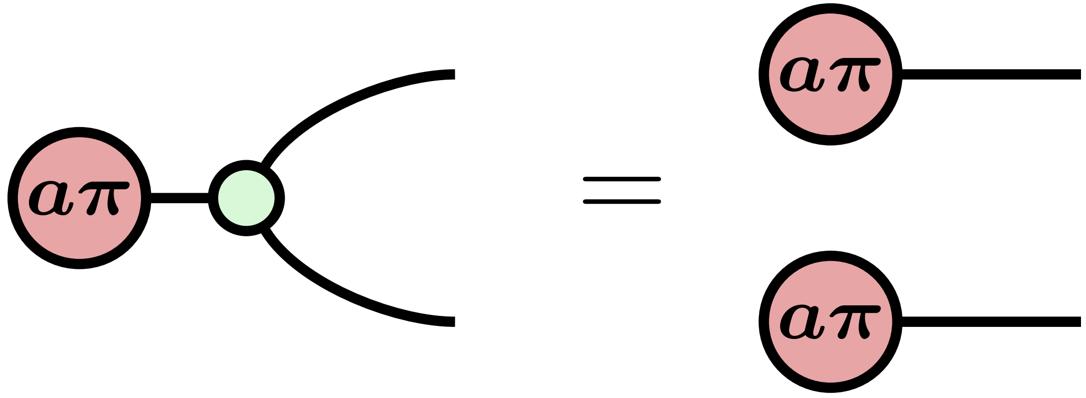
Two ZX calculus rules

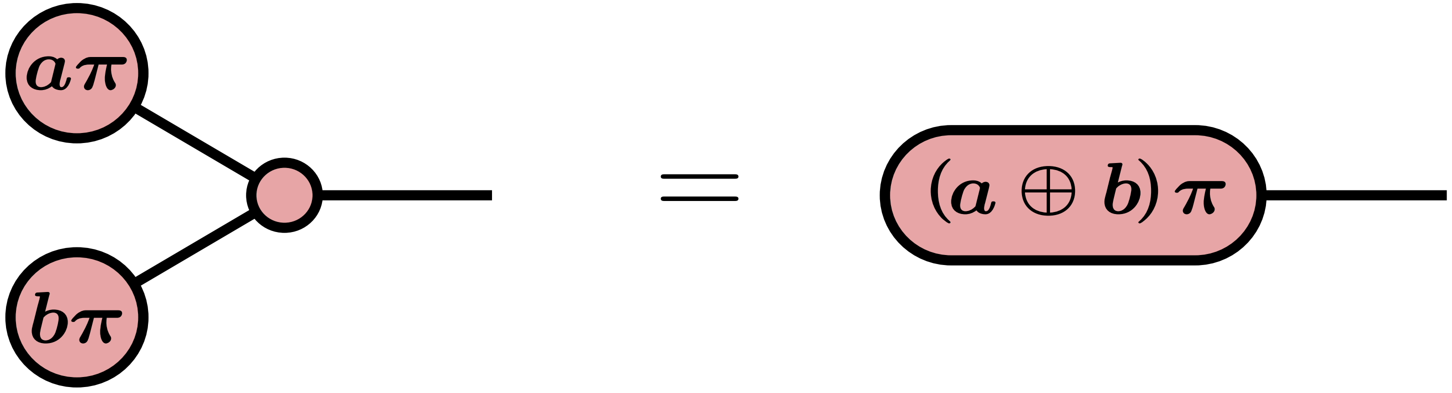




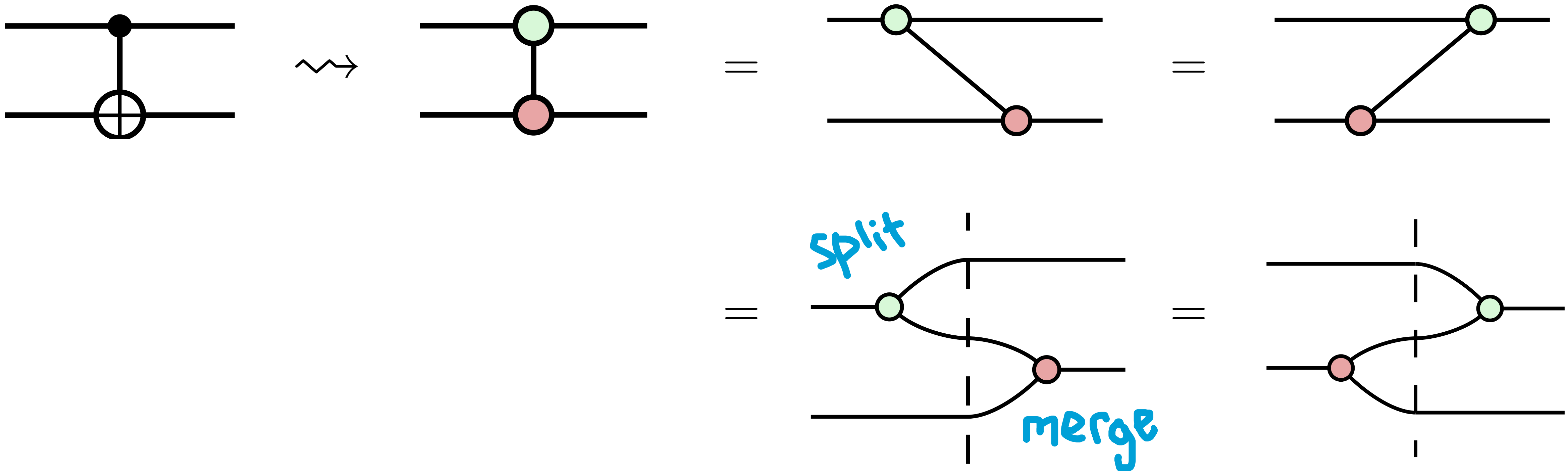
Copy & addition

$a, b \in \{0,1\}$



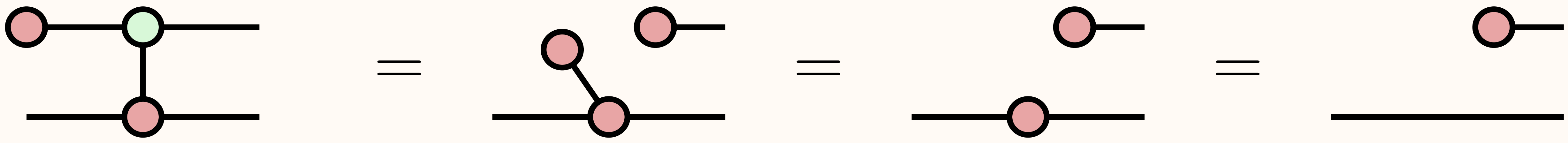


CX gate

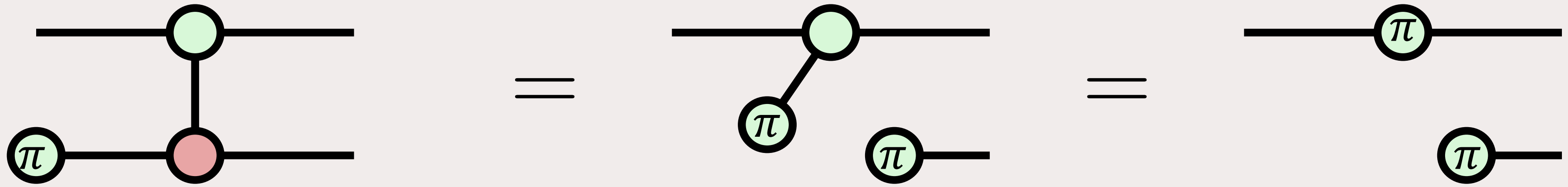


This is how CX is logically implemented in the surface code.

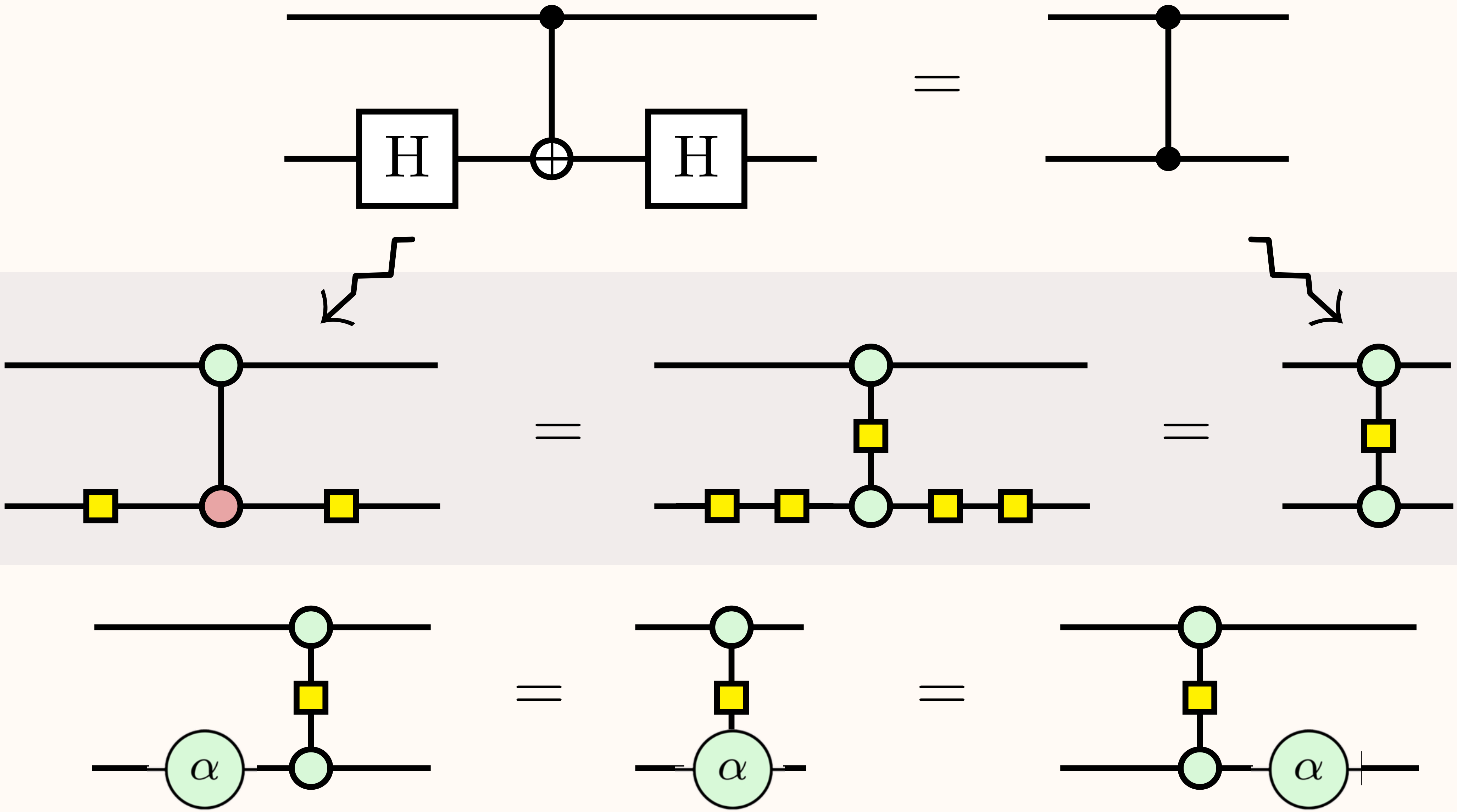
CX with control $|0\rangle$

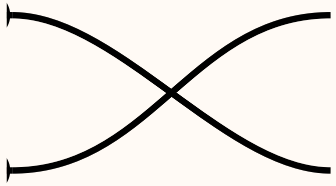


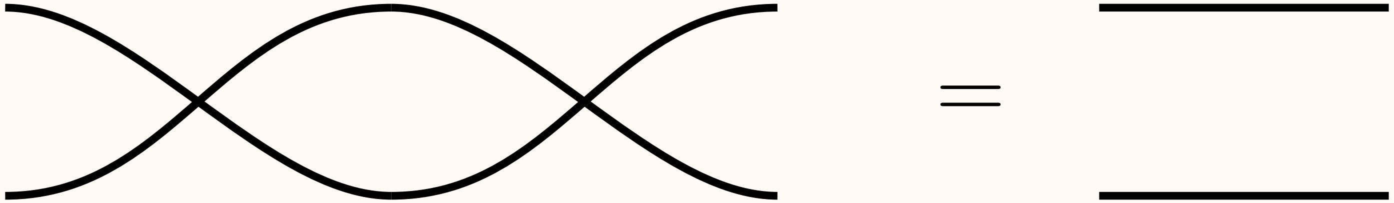
CX with target $|-\rangle$

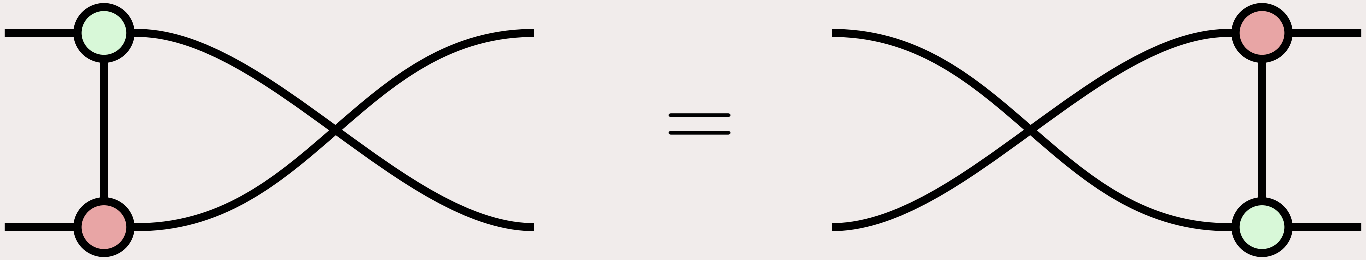


CZ gate

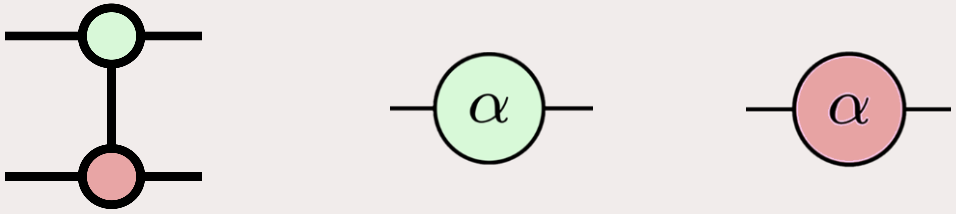


SWAP gate 



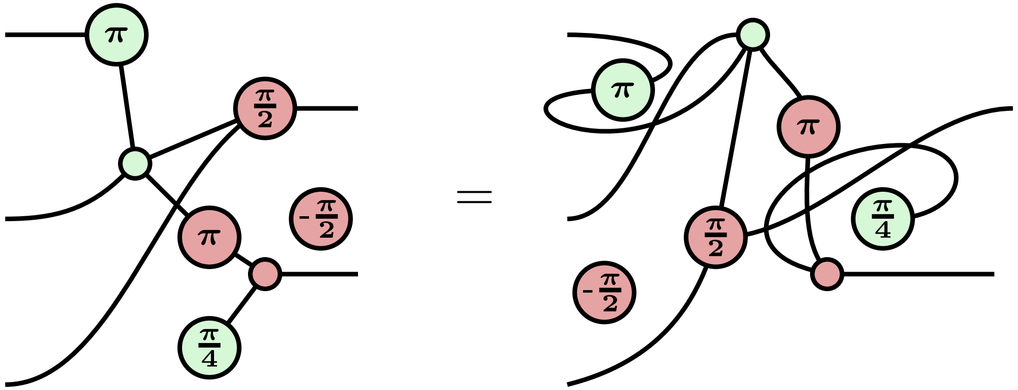


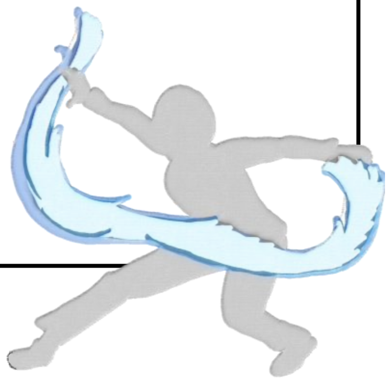
The Clifford+Phases Universal Gate Set



Any circuits made of one-qubit Z basis states, these gates, and one-qubit measurements and post-selections is universal for qubit computation.

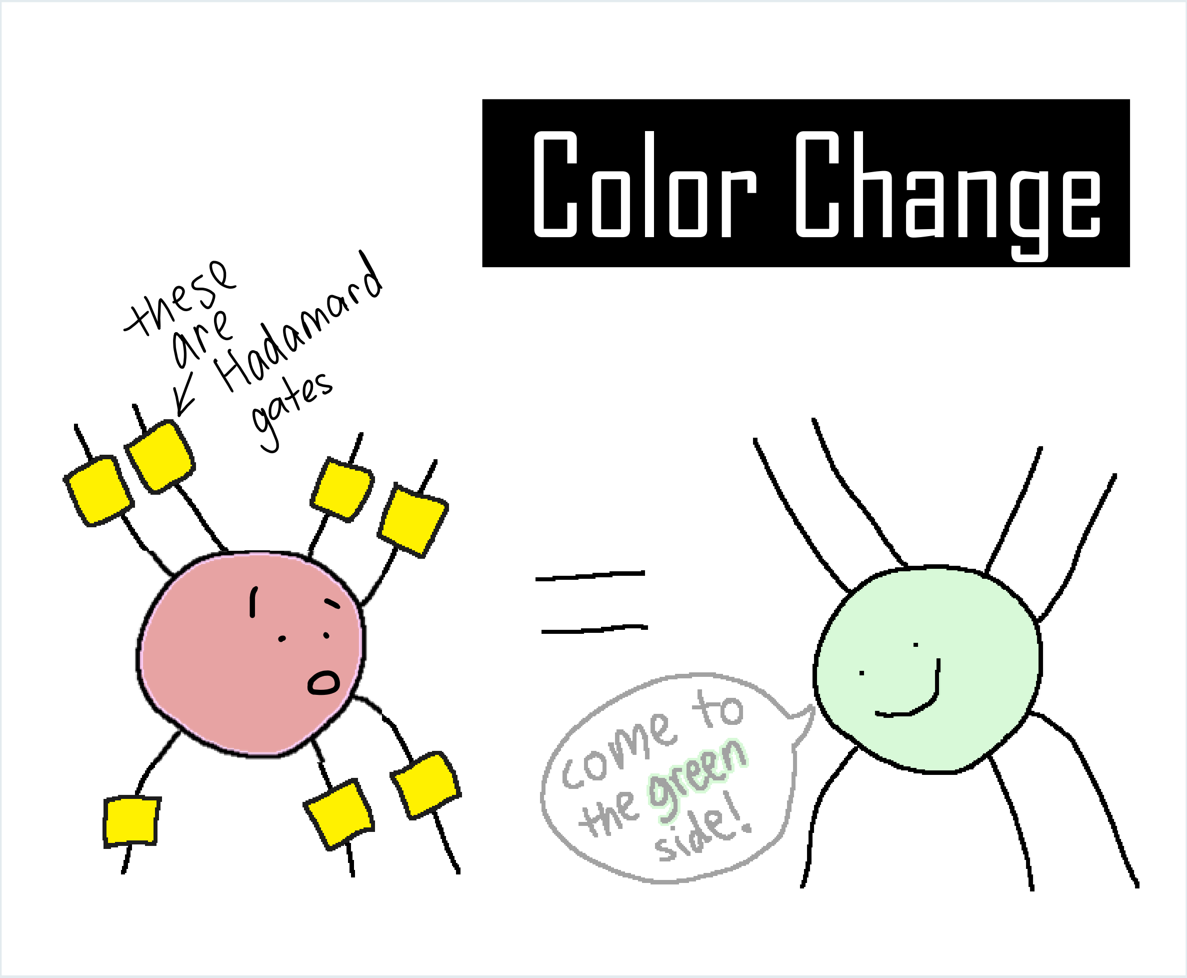
Only Connectivity Matters



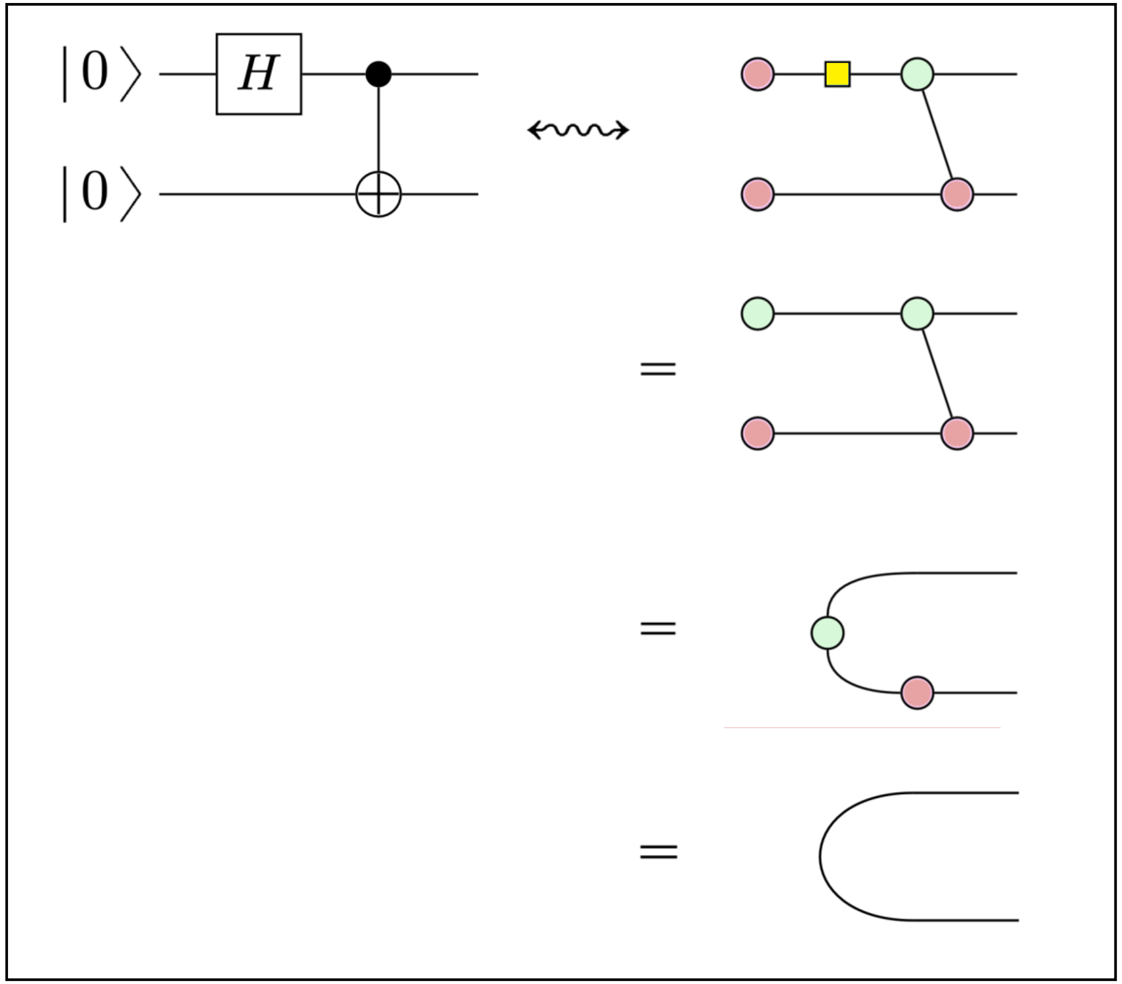


<u>Universality</u>:

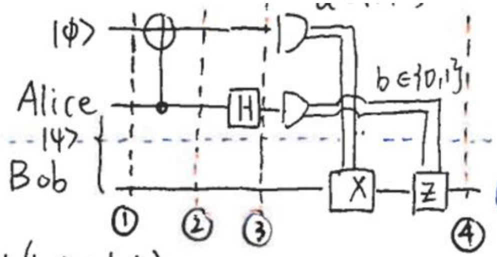
These diagrams suffice to represent any linear map on any number of qubits.


Color Change
these are Hadamard gates
come to the green side!

Bell State


$|0\rangle$
H
$|0\rangle$
$=$
$=$
$=$

Quantum teleportation in bra-ket notation



$$|\phi\rangle^{(1)} = \frac{1}{2\sqrt{2}}(|00\rangle+|11\rangle)(\alpha|0\rangle+\beta|1\rangle) + \frac{1}{2\sqrt{2}}(|00\rangle-|11\rangle)(\alpha|0\rangle-\beta|1\rangle) +$$
$$\frac{1}{2\sqrt{2}}(|01\rangle+|10\rangle)(\alpha|1\rangle+\beta|0\rangle) + \frac{1}{2\sqrt{2}}(|01\rangle-|10\rangle)(\alpha|1\rangle-\beta|0\rangle)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$

$$|\phi\rangle = \alpha|0\rangle+\beta|1\rangle, \ \alpha,\beta\in\mathbb{C}$$

Time slice ① $|\phi\rangle|\psi\rangle = (\alpha|0\rangle+\beta|1\rangle)\otimes\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle) = \frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\beta}{\sqrt{2}}|100\rangle + \frac{\beta}{\sqrt{2}}|111\rangle \quad (1)$

Time slice ② $(CNOT_{21}\otimes I)|\phi\rangle|\psi\rangle = \frac{1}{2\sqrt{2}}(|00\rangle+|01\rangle)(\alpha|0\rangle+\beta|1\rangle) + \frac{1}{2\sqrt{2}}(|00\rangle-|01\rangle)(\alpha|0\rangle-\beta|1\rangle) +$

$$\frac{1}{2\sqrt{2}}(|11\rangle+|10\rangle)(\alpha|1\rangle+\beta|0\rangle) + \frac{1}{2\sqrt{2}}(|11\rangle-|10\rangle)(\alpha|1\rangle-\beta|0\rangle)$$

$$= \frac{1}{2}|0\rangle|+\rangle(\alpha|0\rangle+\beta|1\rangle) + \frac{1}{2}|0\rangle|-\rangle(\alpha|0\rangle-\beta|1\rangle) + \frac{1}{2}|1\rangle|+\rangle(\alpha|1\rangle+\beta|0\rangle).$$
$$-\frac{1}{2}|1\rangle|-\rangle(\alpha|1\rangle-\beta|0\rangle) \quad = |\delta\rangle$$

Time slice ③ $(I\otimes H\otimes I)|\delta\rangle = \frac{1}{2}|0\rangle|0\rangle(\alpha|0\rangle+\beta|1\rangle) + \frac{1}{2}|0\rangle|1\rangle(\alpha|0\rangle-\beta|1\rangle + \frac{1}{2}|1\rangle|0\rangle(\alpha|1\rangle+\beta|0\rangle)\ \frac{1}{2}|1\rangle|1\rangle(\alpha|1\rangle-\beta|0$

Time slice ④: Case 1: $\langle 00|$ Outcome $\begin{cases} a=0 \\ b=0 \end{cases}$ $Z^0X^0\ (\alpha|0\rangle+\beta|1\rangle) = |\phi\rangle.$

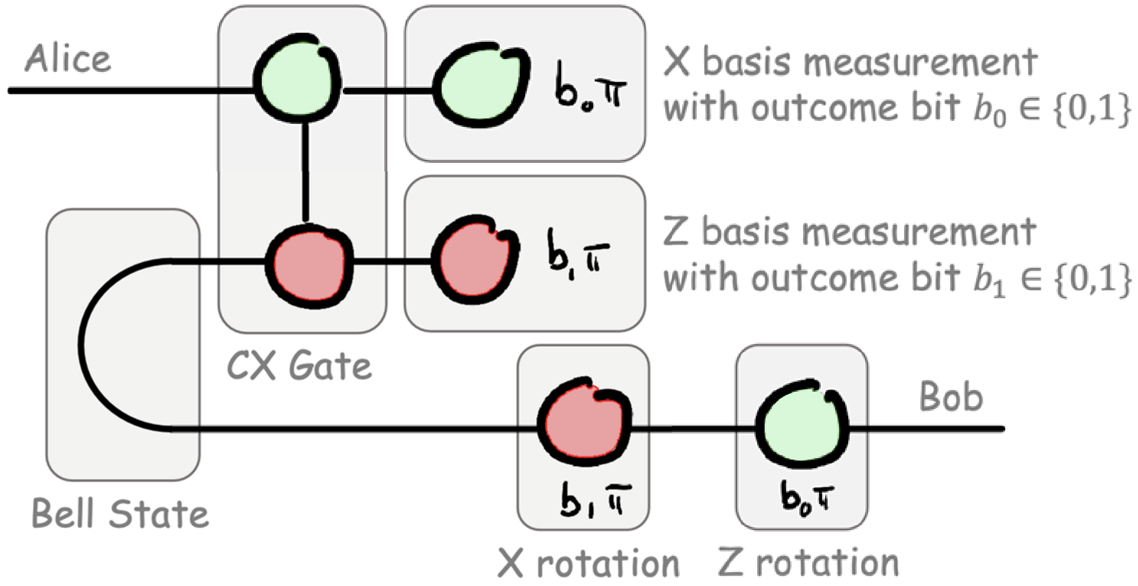
Case 2: $\langle 01|$ Outcome $\begin{cases} a=0 \\ b=1 \end{cases}$ $Z^1X^0(\alpha|0\rangle-\beta|1\rangle) = \alpha|0\rangle+\beta|1\rangle = |\phi\rangle$

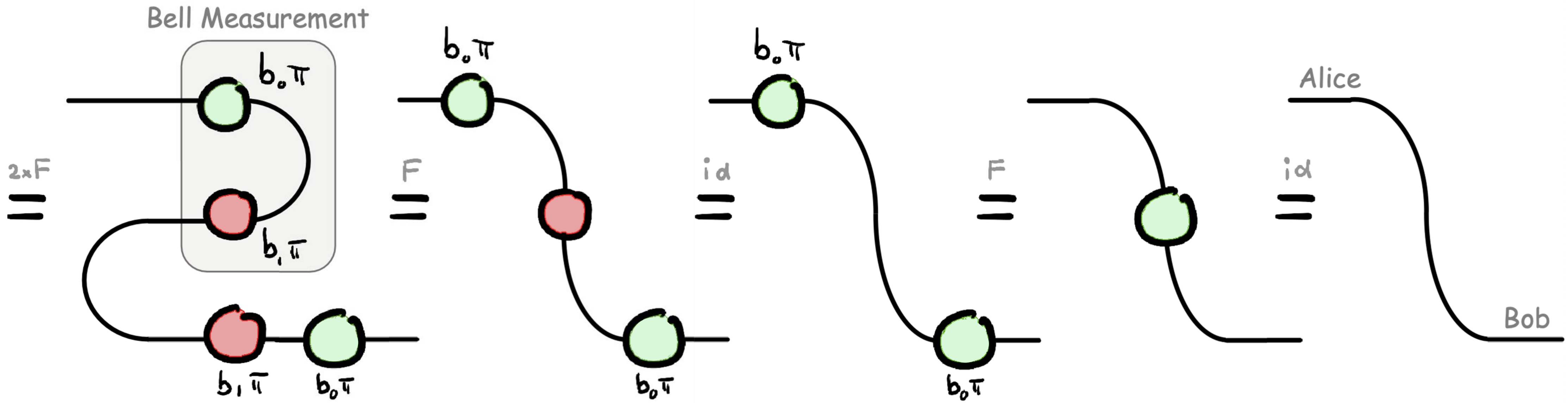
Case 3: $\langle 10|$ Outcom $\begin{cases} a=1 \\ b=0 \end{cases}$ $Z^0X^1(\alpha|1\rangle+\beta|0\rangle) = \alpha|0\rangle+\beta|1\rangle = |\phi\rangle$

Case 4: $\langle 11|$ Outcome $\begin{cases} a=1 \\ b=1 \end{cases}$ $Z^1X^1(\alpha|1\rangle-\beta|0\rangle) = \alpha|0\rangle+\beta|1\rangle = |\phi\rangle$

Student's notes of University of Waterloo's graduate course, courtesy of Sarah Meng Li

Quantum teleportation in ZX calculus

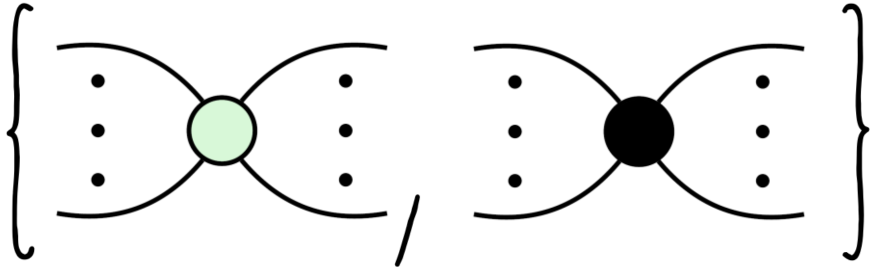




The Completeness Question

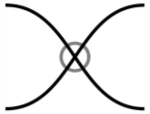
Does proving any equalities
of qubit linear maps
require infinitely many rules?

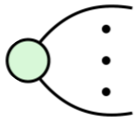
ZW Calculus

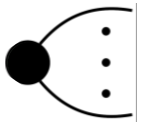


 $= |1\rangle\langle 11| + |0\rangle\langle 01| + |0\rangle\langle 10|$

 $= |00\rangle\langle 0| + |01\rangle\langle 1| + |10\rangle\langle 1|$

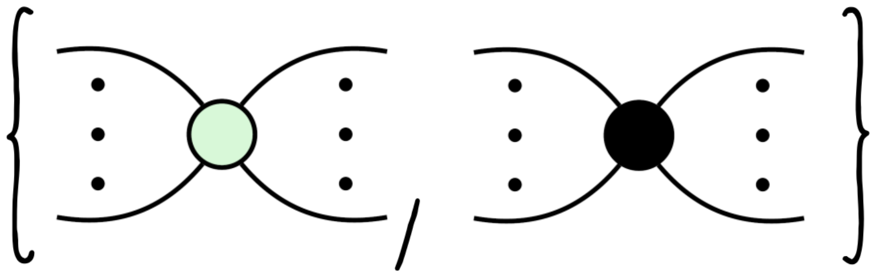
 $= |00\rangle\langle 00| + |01\rangle\langle 10| + |10\rangle\langle 01| - |11\rangle\langle 11|$

 $= |GHZ_N\rangle = |00\ldots 0\rangle + |11\ldots 1\rangle$

 $= |W_N\rangle = |10\ldots 0\rangle + |01\ldots 0\rangle + \ldots + |0\ldots 01\rangle$

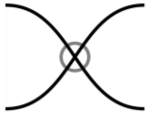
The Compositional Structure of Multipartite Quantum Entanglement. ICALP 2010. Coecke and Kissinger.

ZW Calculus



$$= |1\rangle\langle 11| + |0\rangle\langle 01| + |0\rangle\langle 10|$$

$$= |00\rangle\langle 0| + |01\rangle\langle 1| + |10\rangle\langle 1|$$



$$= |00\rangle\langle 00| + |01\rangle\langle 10| + |10\rangle\langle 01| - |11\rangle\langle 11|$$

In 2017, using ZW-calculus:

Completeness:

You can prove any equalities of qubit linear maps using **just finitely many rules.**

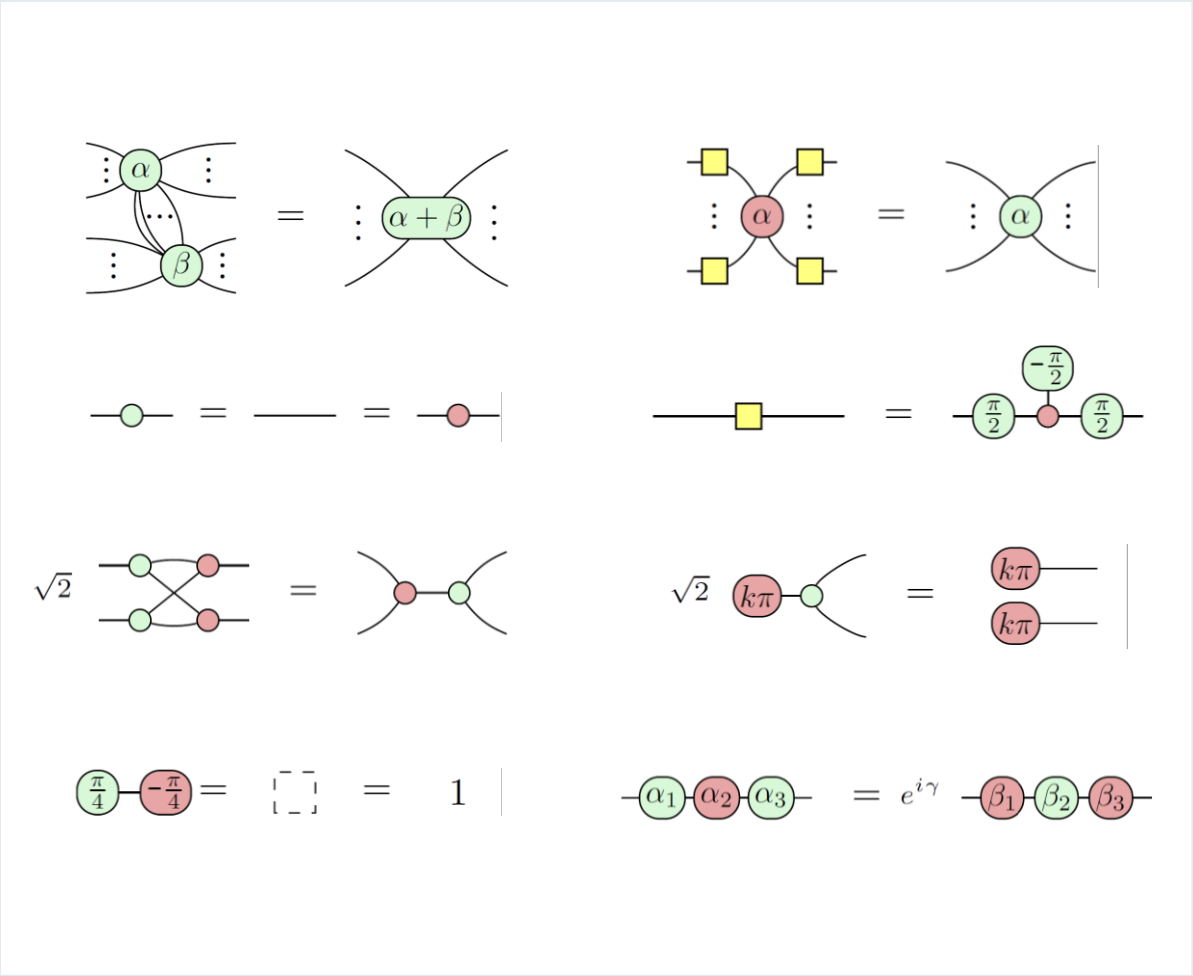
The algebra of entanglement and the geometry of composition. PhD thesis, University of Oxford (2017). Hadzihasanovic.

In 2018, lifted to ZX-calculus:

<u>Completeness</u>:

You can prove any equalities of qubit linear maps using **just finitely many rules.**

Two Complete Axiomatisations of Pure-state Qubit Quantum Computing. LICS (2018). Hadzihasanovic, Ng and Wang.
Diagrammatic Reasoning beyond Clifford+T Quantum Mechanics. LICS (2018). Jeandel, Perdrix, and Vilmart.



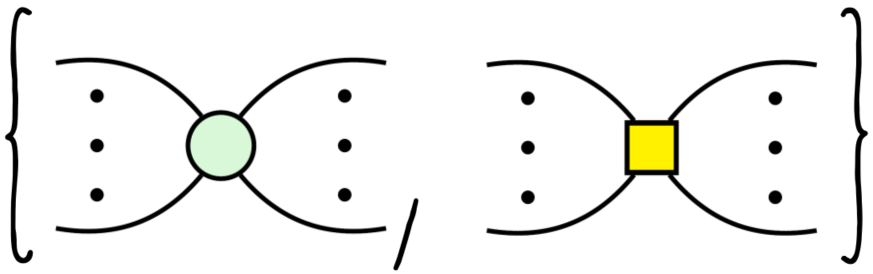
In 2018:

Completeness:

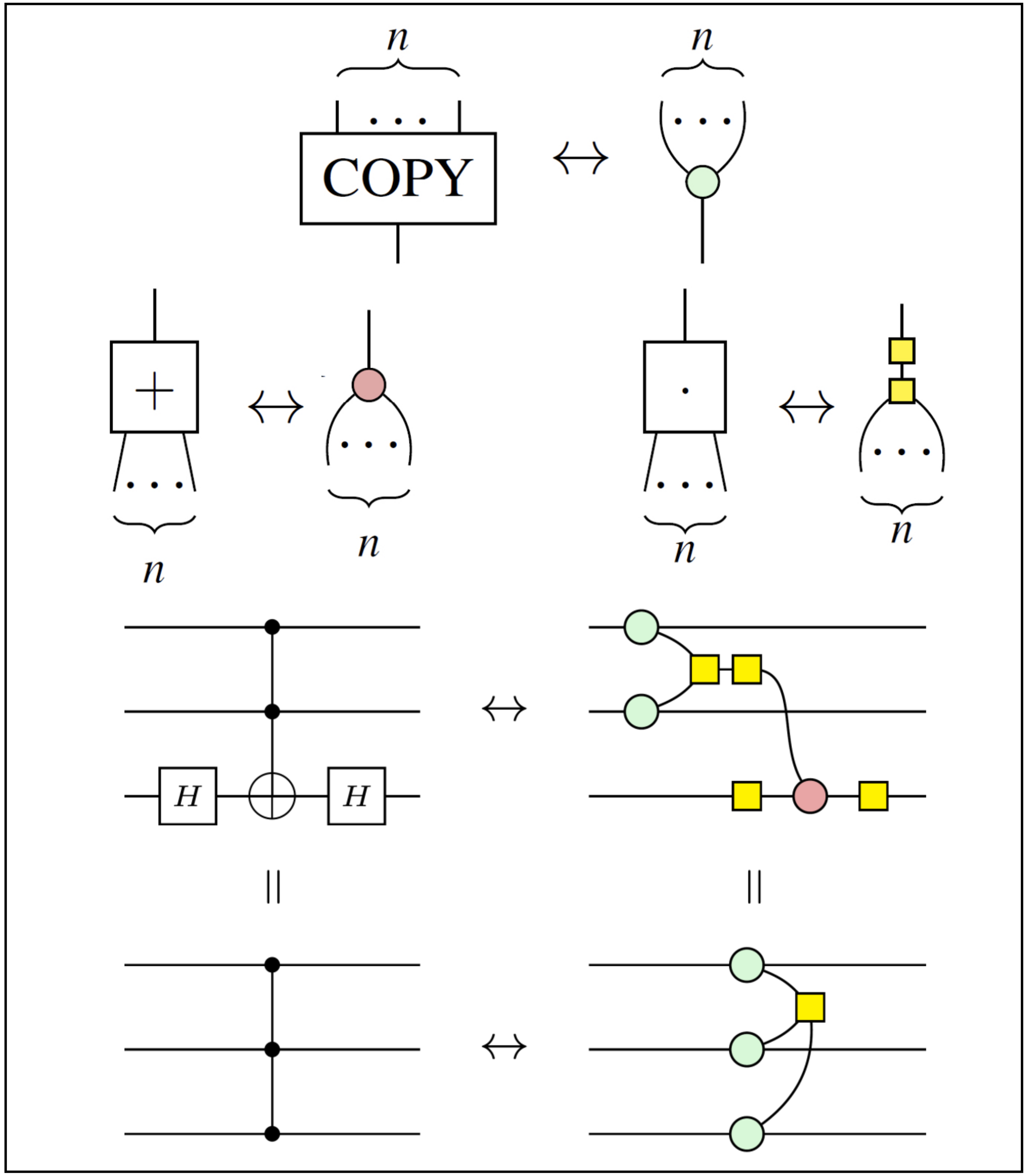
You can prove any equalities of qubit linear maps using just these 8 rules.

A Near-Minimal Axiomatisation of ZX-Calculus for Pure Qubit Quantum Mechanics. LICS (2019). Renaud Vilmart.

ZH Calculus



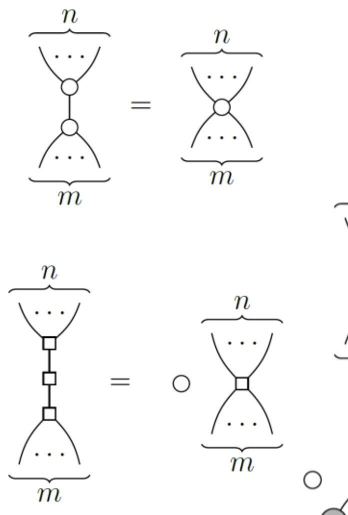
You can draw any Toffoli+Hadamard quantum circuit using just these two types of spiders.

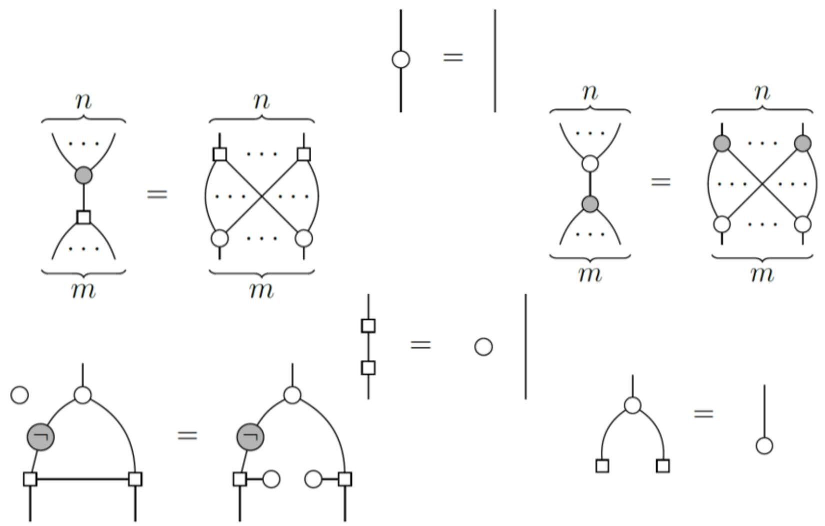


Universality:

These suffice to represent any Toffoli+Hadamard linear map on any number of qubits.

Completeness





You can prove any equalities of qubit Toffoli+Hadamard linear maps using just these 8 rules.

Completeness of the ZH-calculus. Compositionality (2023). Backens, Kissinger, Miller-Bakewell, van de Wetering, Wolffs.

	Qubit Completeness Results
Stabilizer aka Clifford	
Approx. Universal aka Clifford + T	
Universal	· Ng, Wang & Hadzihasanovic, 2018 · Jeandel, Perdrix & Vilmart, 2018

Overview

- Part 1 of the tutorial covered:
  - Process theories and ZX-calculus
  - **Circuit Optimization and Classical Simulation**
  - Measurement-based quantum computation

- Part 2 will cover:
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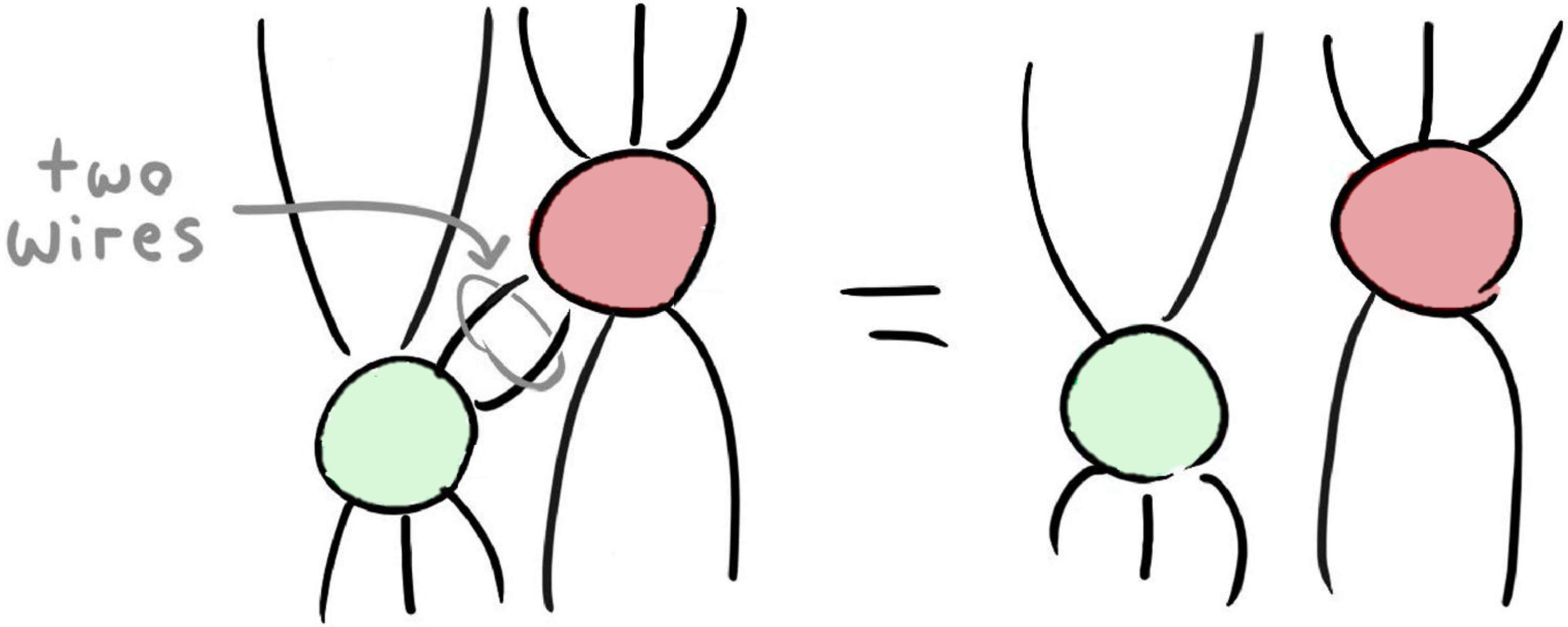
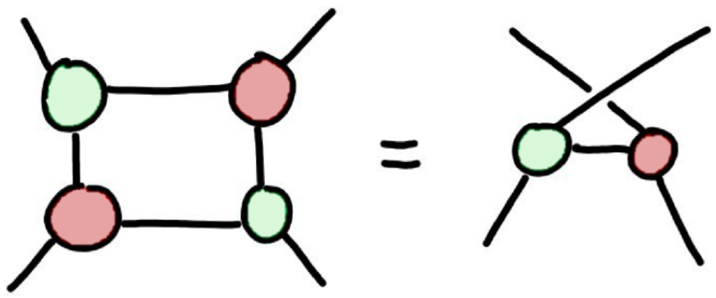
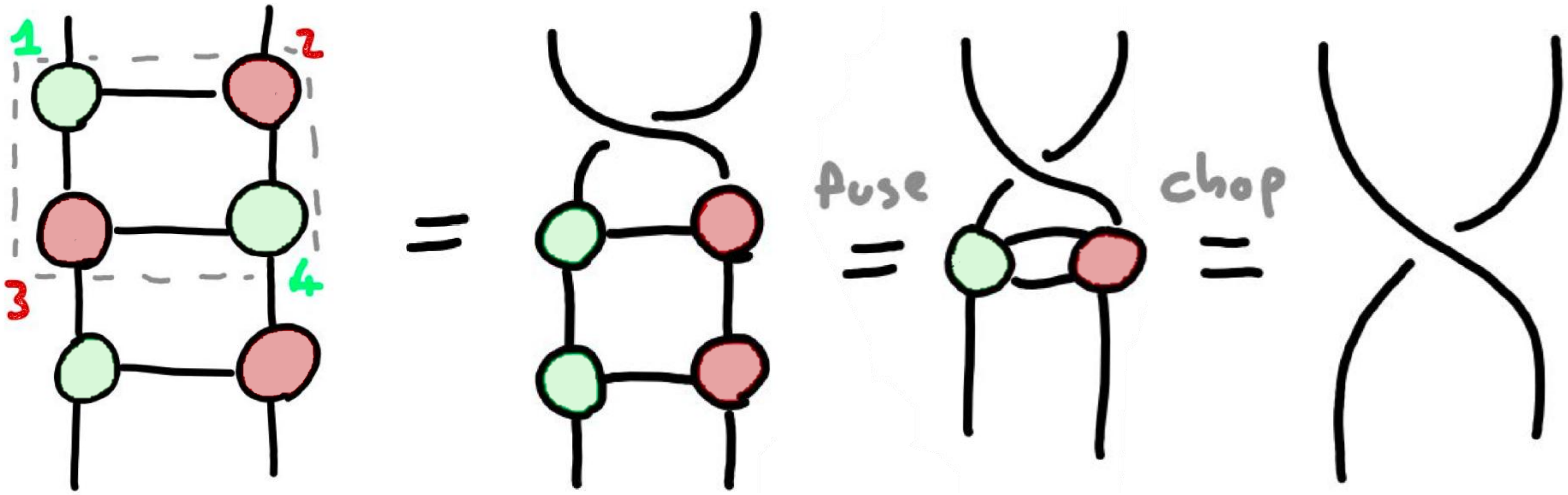
  - Higher dimensional variants of ZX calculus

Hopf rule

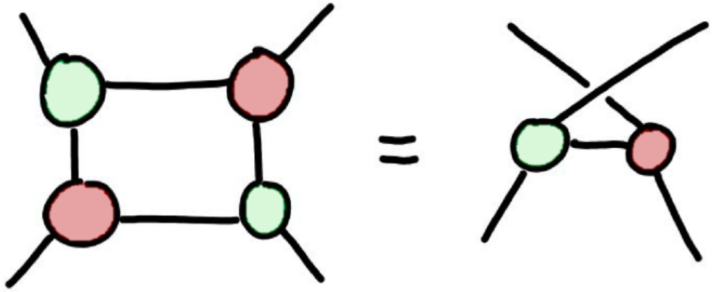


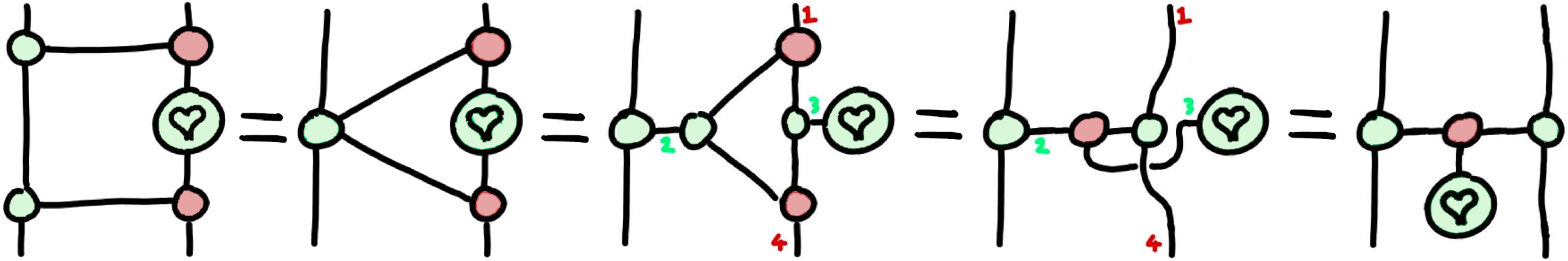
Bialgebra rule

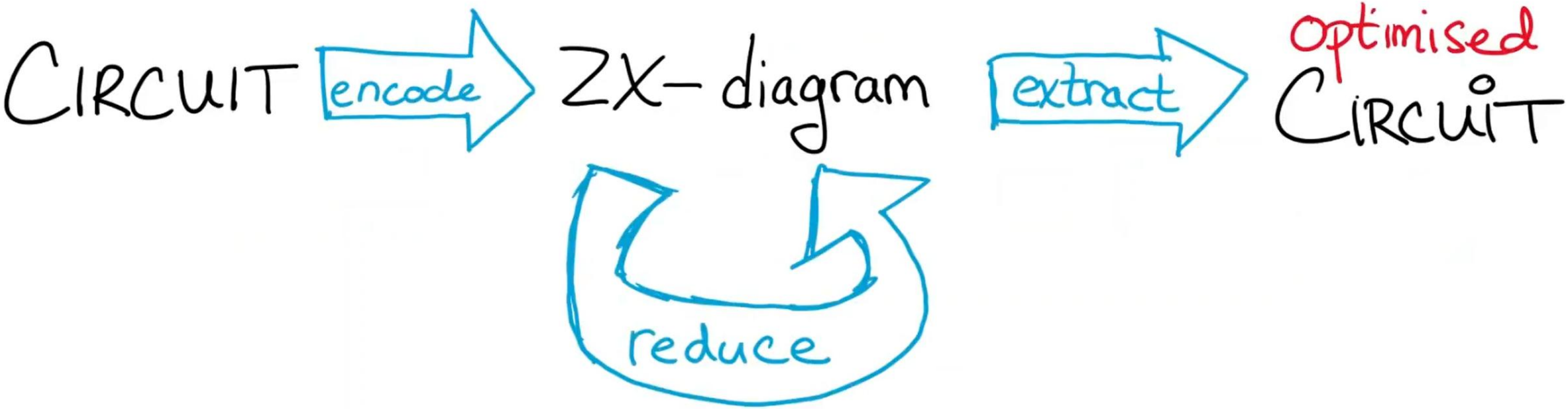




Bialgebra rule

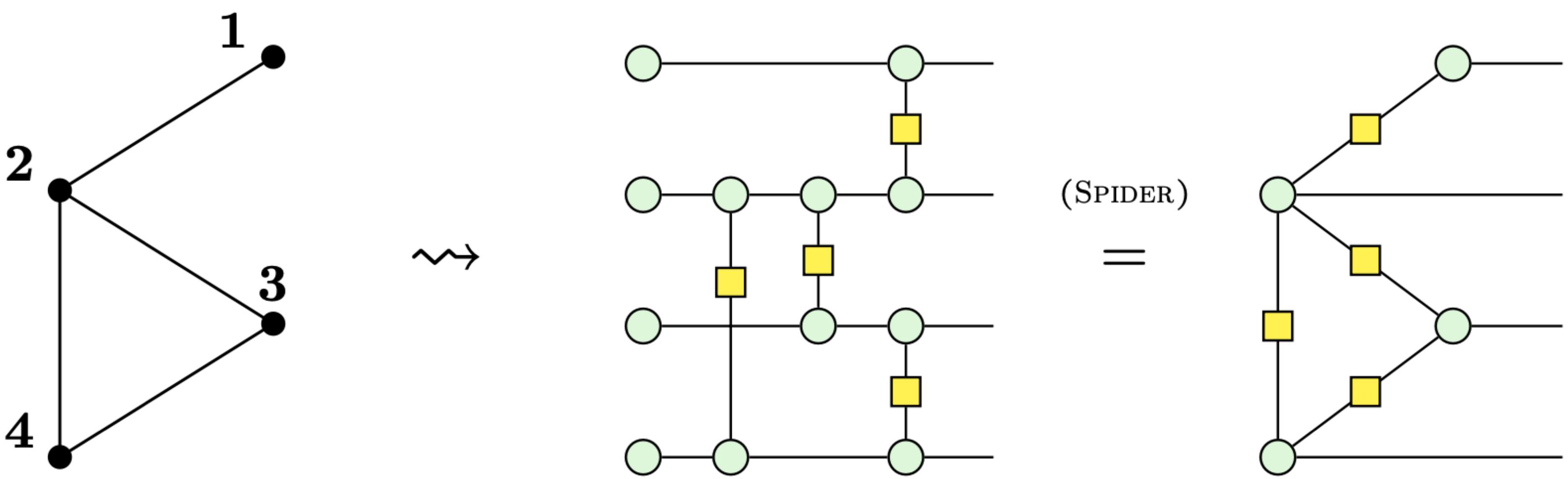




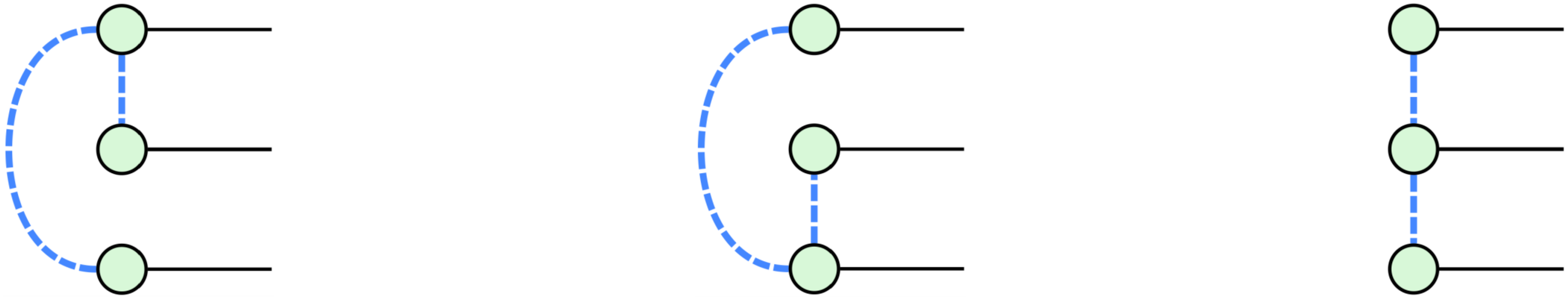


PyZX: Large Scale Automated Diagrammatic Reasoning. QPL 2019. Kissinger and van de Wetering.

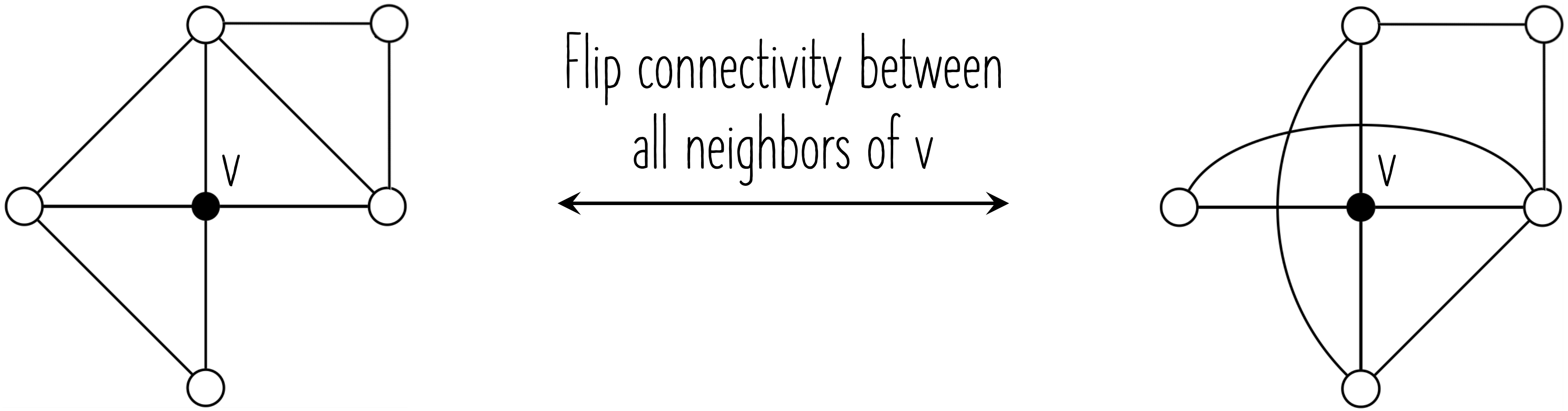
Graph states i.e. $\prod CZ \ket{+}^{\otimes}$



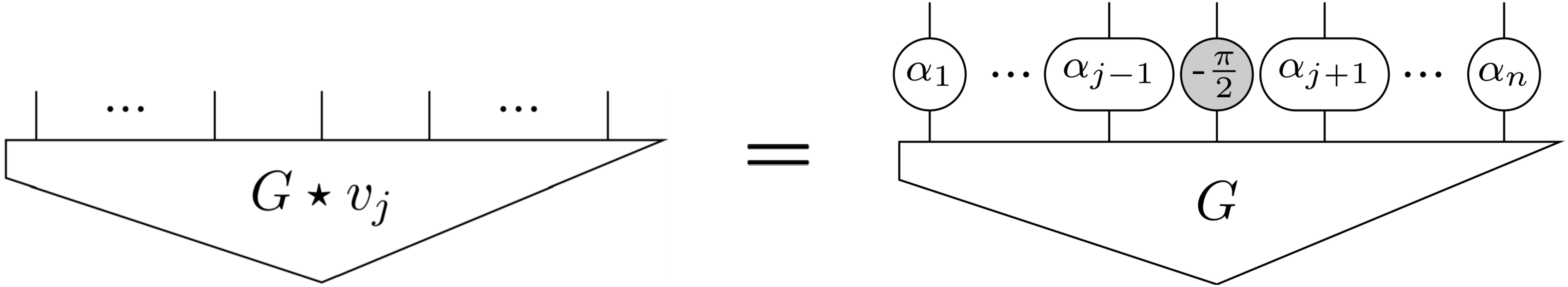
These graph states are equivalent up to local (one-qubit) Cliffords:



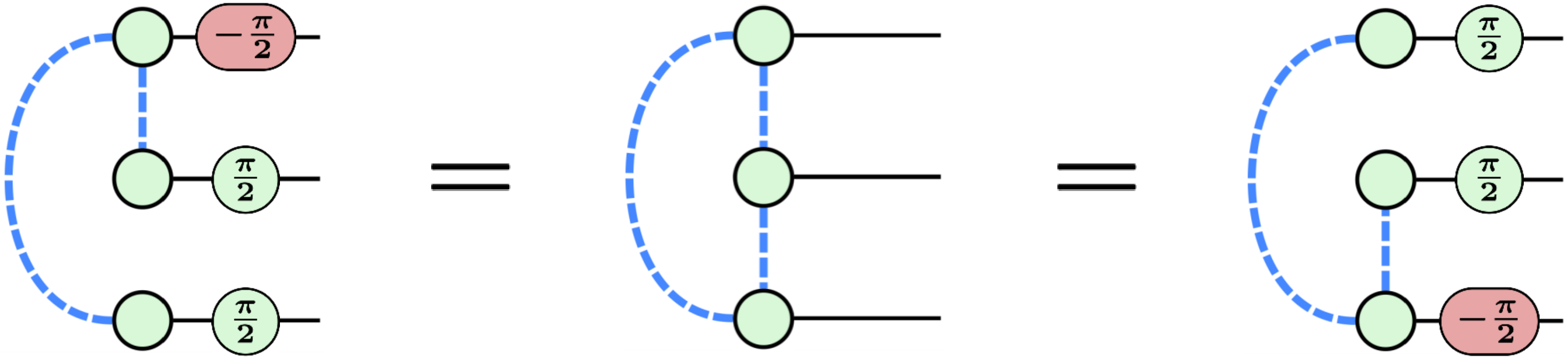
The reason is <u>local complementation</u> $G \star v$:



In general, where $\alpha_i = \begin{cases} \frac{\pi}{2} & \text{if } v_i \text{ is adjacent to } v_j \\ 0 & \text{otherwise} \end{cases}$:



Example:



For all Clifford ZX diagrams, meaning all phases are 0, π, or $\pm\frac{\pi}{2}$:

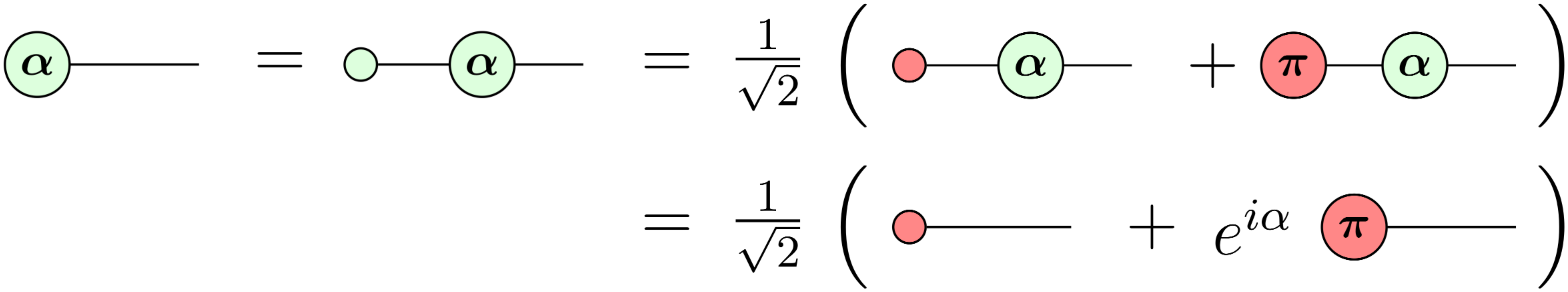
- You can always obtain normal form, polytime in diagram size
  Graph-theoretic Simplification of Quantum Circuits with the ZX-calculus. Quantum (2020). Duncan, Kissinger, Perdrix, and van de Wetering.
  - Moreover, admits a unique normal form
    Complete Flow-Preserving Rewrite Rules for MBQC Patterns with Pauli Measurements. QPL 2022. McElvanney and Backens.
    Picturing Quantum Software (textbook preprint). Kissinger and van de Wetering.

- Thus, for any Clifford ZX diagrams, you can always efficiently check if same linear map, by checking if unique normal forms are same

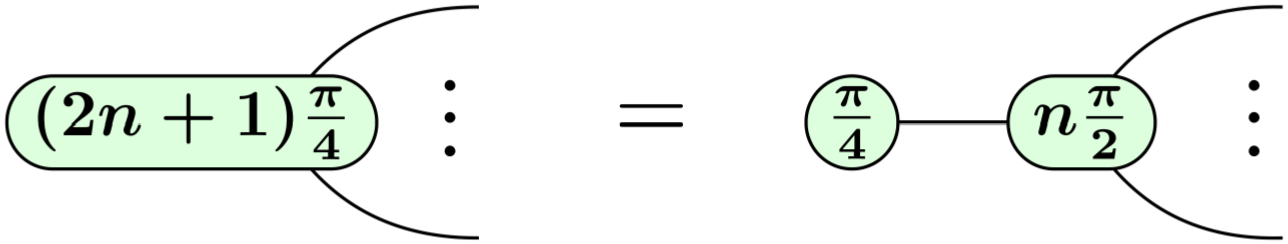
General strategy for any ZX diagram:
Unfuse all non-Clifford phases,
then optimize the Clifford subdiagram

	Qubit Completeness Results
Stabilizer aka Clifford	· Backens, 2014
Approx. Universal aka Clifford + T	· Jeandel, Perdrix & Vilmart, 2018
Universal	· Ng, Wang & Hadzihasanovic, 2018 · Jeandel, Perdrix & Vilmart, 2018

Stabilizer Decompositions

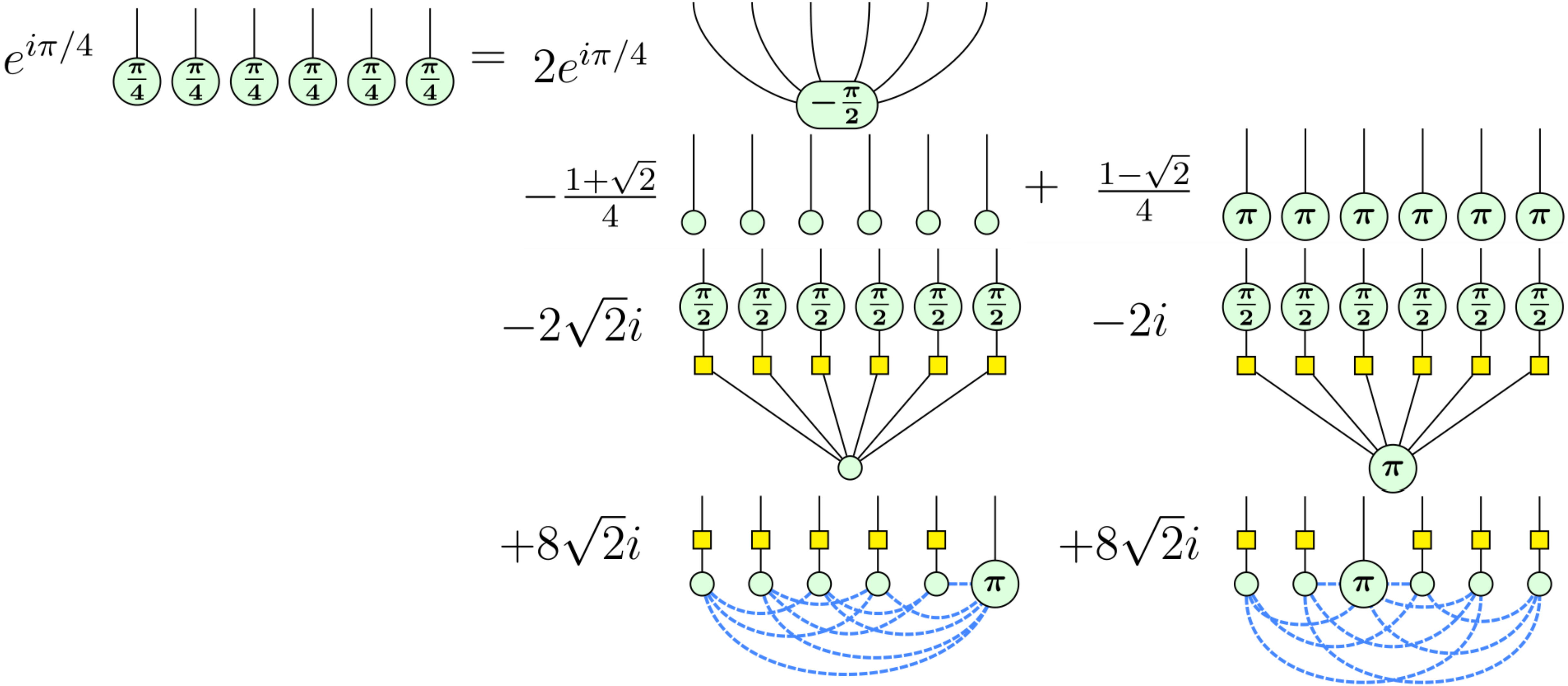


Classically simulate Clifford+T ZX diagrams as a sum of 2^t Clifford terms



More efficient: 6 Ts to 7 Cliffords, i.e. $7^{t/6} \approx 2^{0.47t}$ Clifford terms

Trading Classical and Quantum Computational Resources. PRX Quantum (2016). Bravyi, Smith, Smolin.



Simulating quantum circuits with zx-calculus reduced stabiliser decompositions. Quantum Science and Technology (2022). Kissinger, van de Wetering.

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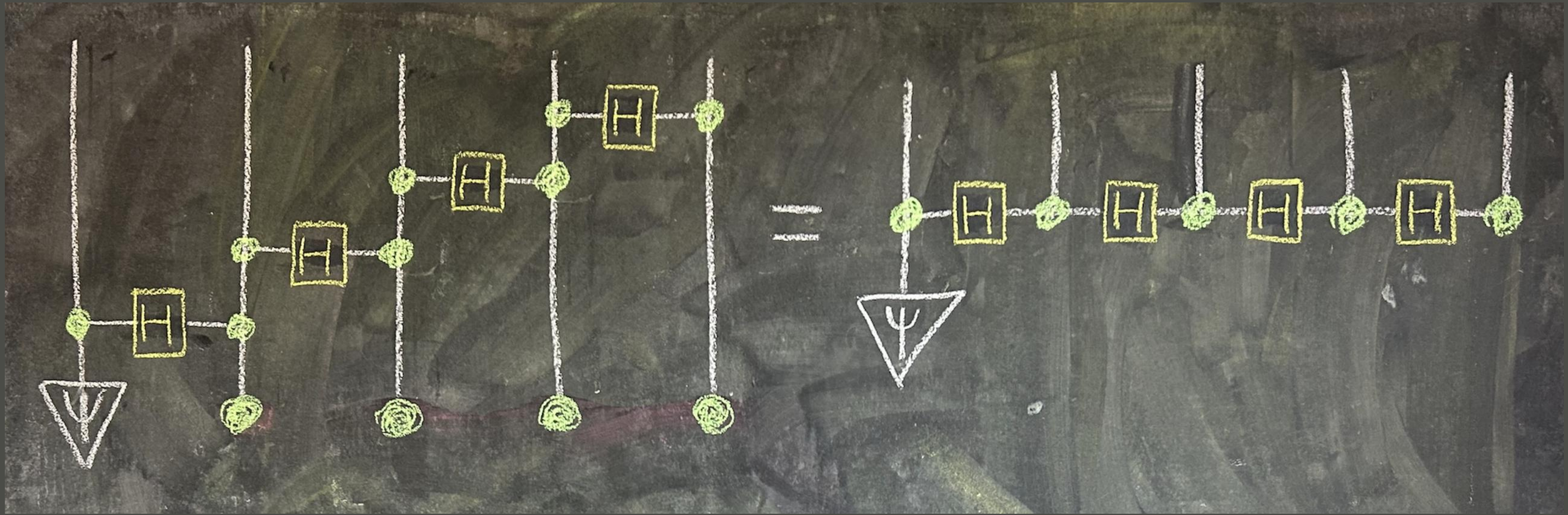
Circuit-Based Quantum Computing (CBQC)

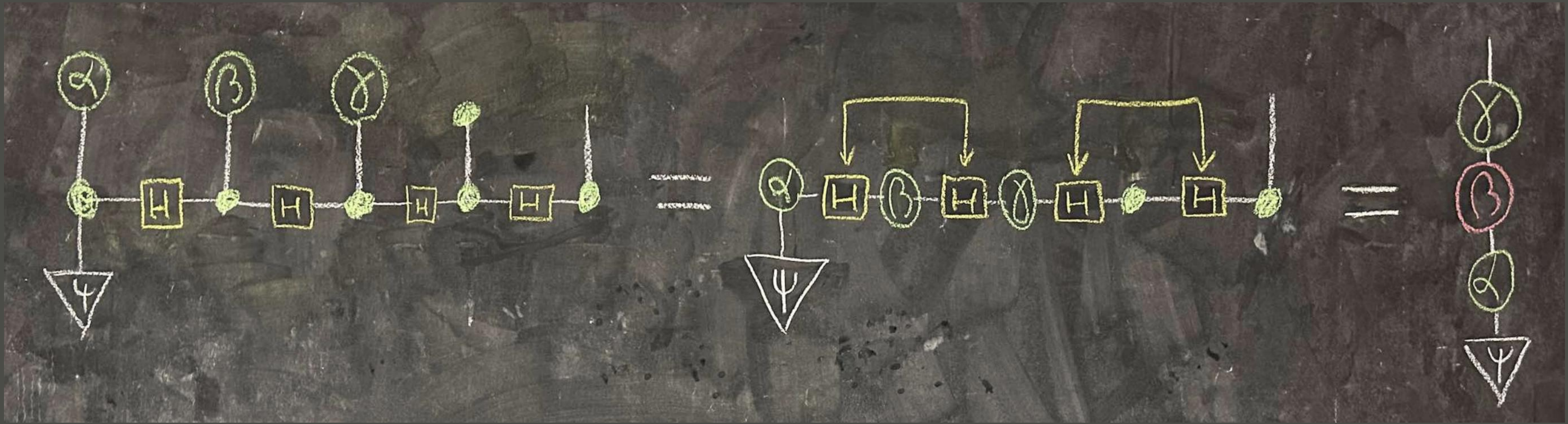
states -> entangling gates -> measure

Measurement-Based Quantum Computing (MBQC)

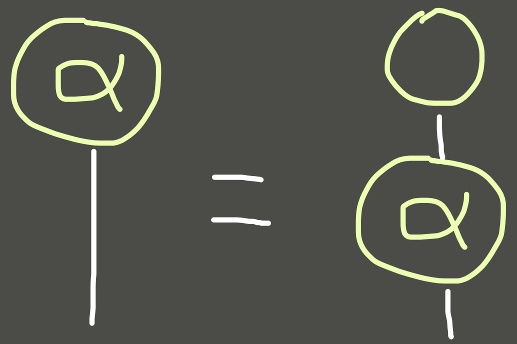
entangled resource states -> measure -> correct based on past measurements

A One-Way Quantum Computer. PRL (2001). Raussendorf and Briegel.

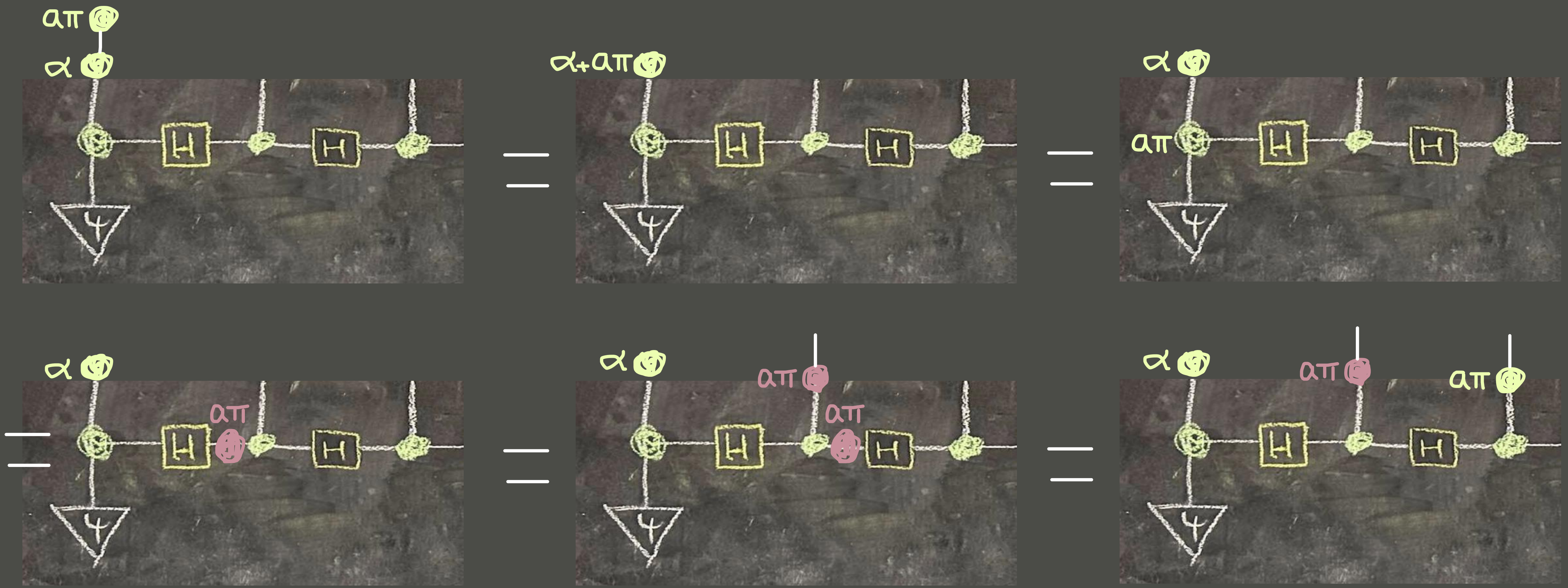




From Quantum in Pictures (course lecture videos) at quantuminpictures.org

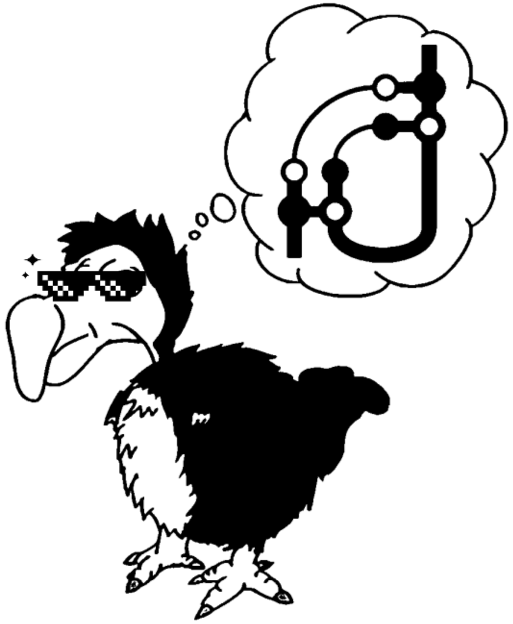


Can't be implemented deterministically. So let $a \in \{0,1\}$



to be continued

part 2 coming to a screen near you next



Thank you for listening